Cuaderno de notas de trabajo

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Cuaderno 30

$$\left.\begin{array}{l}\text{1 unidad de masa}\\ \text{Birkhoffiana}\end{array}\right\} = 4.0435 \cdot 10^{38}\ \text{gr.}$$

1 gramo $= 2.4731 \cdot 10^{-39}$ Unidades Birkhoffianas

Masa del Sol $=$ ~~$4.9462 \cdot 10^{-6}$ UB~~

$4.874 \cdot 10^{-6}$ UB

Seudocurvatura del campo en la teoría de la gravitación de Birkhoff.

El tensor de curvatura en un espacio Riemanniano es:

$$1) \quad R^{a\cdots}_{\cdot ijk} = \frac{\partial}{\partial x^j}\Gamma^a_{ik} - \frac{\partial}{\partial x^k}\Gamma^a_{ij} + \Gamma^a_{bj}\Gamma^b_{ik} - \Gamma^a_{bk}\Gamma^b_{ij}.$$

En donde Γ^a_{ik} es el símbolo de Christoffel.

En la teoría de Birkhoff las ~~trayect~~ líneas de universo de las partículas ~~son~~ son senderos definidos por:

$$\cancel{\frac{d^2x^i}{ds^2} + \Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0}$$

$$2) \quad \frac{d^2x^i}{ds^2} + B^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

En donde $B^i_{jk} = \cancel{\Delta^i_{\ell m}}\,\Delta^{im}\left(\frac{\partial h_{mj}}{\partial x^k} - \frac{\partial h_{ij}}{\partial x^m}\right)$

La ~~pseudo cu~~ seudocurvatura del campo Birkhoffiano es entonces (por

La ecuación diferencial de las líneas de universo ~~de~~ en la teoría de Birkhoff es:

$$\frac{d^2x^i}{ds^2} + \Delta^{im}\left(-\frac{\partial h_{mj}}{\partial x^k} + \frac{\partial h_{jk}}{\partial x^m}\right)\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

$$\frac{d^2x^i}{ds^2} + \Delta^{im}\left[\frac{1}{2}\left(-\frac{\partial h_{mj}}{\partial x^k} - \frac{\partial h_{mk}}{\partial x^j} + 2\frac{\partial h_{jk}}{\partial x^m}\right)\right]\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

$$\frac{d^2x^i}{ds} + B^{i\cdot\cdot}_{\cdot jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

$$B^{i\cdot\cdot}_{\cdot jk} = \frac{1}{2}\Delta^{im}\left[2\frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{mj}}{\partial x^k} - \frac{\partial h_{mk}}{\partial x^j}\right]$$

El tensor de seudocurvatura del campo en la teoría de la gravitación de Birkhoff es:

$$R^{a\cdot\cdot\cdot}_{\cdot ijk} = \frac{\partial}{\partial x^j}B^{a\cdot\cdot}_{\cdot ik} - \frac{\partial}{\partial x^k}B^{a\cdot\cdot}_{\cdot ij} + B^{a\cdot\cdot}_{\cdot bj}B^{b\cdot\cdot}_{\cdot ik} - B^{a\cdot\cdot}_{\cdot bk}B^{b\cdot\cdot}_{\cdot ij}$$

$$\frac{\partial}{\partial x^j}B^{a\cdot\cdot}_{\cdot ik} = \Delta^{am}\frac{1}{2}\left[2\frac{\partial^2 h_{ik}}{\partial x^j\partial x^m} - \cancel{\frac{\partial^2 h_{mi}}{\partial x^j\partial x^k}} - \frac{\partial^2 h_{mk}}{\partial x^i\partial x^j}\right]$$

$$-\frac{\partial}{\partial x^k}B^{a\cdot\cdot}_{\cdot ij} = \Delta^{am}\frac{1}{2}\left[-2\frac{\partial^2 h_{ij}}{\partial x^k\partial x^m} + \cancel{\frac{\partial^2 h_{mi}}{\partial x^k\partial x^j}} + \frac{\partial^2 h_{mj}}{\partial x^k\partial x^i}\right]$$

$$B^{a\cdot\cdot}_{\cdot bj}B^{b\cdot\cdot}_{\cdot ik} = \frac{1}{4}\Delta^{am}\Delta^{bn}\left[2\frac{\partial h_{bj}}{\partial x^m} - \frac{\partial h_{mb}}{\partial x^j} - \frac{\partial h_{mj}}{\partial x^b}\right]\left[2\frac{\partial h_{ik}}{\partial x^n} - \frac{\partial h_{ni}}{\partial x^k} - \frac{\partial h_{nk}}{\partial x^i}\right]$$

$$-B^{a\cdot\cdot}_{\cdot bk}B^{b\cdot\cdot}_{\cdot ij} = -\Delta^{am}\Delta^{bn}\frac{1}{4}\left[2\frac{\partial h_{bk}}{\partial x^m} - \frac{\partial h_{mb}}{\partial x^k} - \frac{\partial h_{mk}}{\partial x^b}\right]\left[2\frac{\partial h_{ij}}{\partial x^n} - \frac{\partial h_{ni}}{\partial x^j} - \frac{\partial h_{nj}}{\partial x^i}\right]$$

$$R^{a\cdot\cdot\cdot}_{\cdot ijk} =$$

$$B^{i\cdot\cdot}_{\cdot jk} = \frac{1}{2}\Delta^{im}\left[\frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{jm}}{\partial x^k} + \frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{km}}{\partial x^j}\right]$$

$$D_m h_{jk} = \frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{jm}}{\partial x^k} + \frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{km}}{\partial x^j}$$

$$v_{m;j} = \frac{\partial v_m}{\partial x^j} - \frac{\partial v_j}{\partial x^m}$$

$$T_{mn;j} = \frac{\partial T_{mn}}{\partial x^j} - \frac{\partial T_{jn}}{\partial x^m} + \frac{\partial T_{mn}}{\partial x^j} - \frac{\partial T_{mj}}{\partial x^m}$$

Transformación del tensor gravitacional de Birkhoff en los cambios de coordenadas

$$\frac{d^2x^i}{ds^2} + B^i_{jk}\frac{dx^i}{ds}\frac{dx^j}{ds} = 0$$

B^i_{jk} — tensor intensidad gravitacional de Birkhoff.

$$\frac{d\bar{x}^i}{ds} = \frac{\partial \bar{x}^i}{\partial x^m}\frac{dx^m}{ds}$$

$$\frac{d^2\bar{x}^i}{ds^2} = \frac{\partial^2 \bar{x}^i}{\partial x^m \partial x^n}\frac{dx^m}{ds}\frac{dx^n}{ds} + \frac{\partial \bar{x}^i}{\partial x^m}\frac{d^2x^m}{ds^2}$$

$$-\bar{B}^i_{jk}\frac{d\bar{x}^j}{ds}\frac{d\bar{x}^k}{ds} = \frac{\partial^2 \bar{x}^i}{\partial x^m \partial x^n}\frac{dx^m}{ds}\frac{dx^n}{ds} + \frac{\partial \bar{x}^i}{\partial x^m}\left(-B^m_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds}\right)$$

$$\begin{aligned}-\bar{B}^i_{jk}\frac{d\bar{x}^j}{ds}\frac{d\bar{x}^k}{ds} = \Big[&\frac{\partial^2 \bar{x}^i}{\partial x^m \partial x^n}\frac{\partial x^m}{\partial \bar{x}^j}\frac{\partial x^n}{\partial \bar{x}^k}\frac{d\bar{x}^j}{ds}\frac{d\bar{x}^k}{ds} \\ &\frac{\partial \bar{x}^i}{\partial x^m}B^m_{pq}\frac{\partial x^p}{\partial \bar{x}^j}\frac{\partial x^q}{\partial \bar{x}^k}\frac{d\bar{x}^j}{ds}\frac{d\bar{x}^k}{ds}\Big]\end{aligned}$$

$$\bar{B}^{i}_{jk} = B^{m}_{pq} \frac{\partial \bar{x}^{i}}{\partial x^{m}} \frac{\partial x^{p}}{\partial \bar{x}^{j}} \frac{\partial x^{q}}{\partial \bar{x}^{k}} - \frac{\partial^{2} \bar{x}^{i}}{\partial x^{m} \partial x^{n}} \frac{\partial x^{m}}{\partial \bar{x}^{j}} \frac{\partial x^{n}}{\partial \bar{x}^{k}}$$

$$\cancel{\partial x} \quad \frac{\partial \bar{x}^{i}}{\partial x^{m}} \frac{\partial x^{m}}{\partial \bar{x}^{j}} = \delta^{i}_{j}$$

$$\frac{\partial}{\partial x^{n}}$$

$$\frac{\partial^{2} \bar{x}^{i}}{\partial x^{m} \partial x^{n}} \frac{\partial x^{m}}{\partial \bar{x}^{j}} + \frac{\partial \bar{x}^{i}}{\partial x^{m}} \frac{\partial^{2} x^{m}}{\partial \bar{x}^{j} \partial \bar{x}^{k}} \frac{\partial \bar{x}^{k}}{\partial x^{n}} = 0$$

$$\bar{B}^{i}_{jk} = B^{m}_{pq} \frac{\partial \bar{x}^{i}}{\partial x^{m}} \frac{\partial x^{p}}{\partial \bar{x}^{j}} \frac{\partial x^{q}}{\partial \bar{x}^{k}} + \frac{\partial^{2} x^{m}}{\partial \bar{x}^{j} \partial \bar{x}^{r}} \frac{\partial \bar{x}^{i}}{\partial x^{m}} \frac{\partial \bar{x}^{r}}{\partial x^{n}} \frac{\partial x^{n}}{\partial \bar{x}^{k}}$$

$$\bar{B}^{i}_{jk} = B^{m}_{pq} \frac{\partial \bar{x}^{i}}{\partial x^{m}} \frac{\partial x^{p}}{\partial \bar{x}^{j}} \frac{\partial x^{q}}{\partial \bar{x}^{k}} + \frac{\partial^{2} x^{m}}{\partial \bar{x}^{j} \partial \bar{x}^{k}} \frac{\partial \bar{x}^{i}}{\partial x^{m}}$$

$$\boxed{\begin{array}{l} \text{Ecuaciones de transformación} \\ \bar{B}^{i}_{jk} = B^{m}_{pq} \frac{\partial \bar{x}^{i}}{\partial x^{m}} \frac{\partial x^{p}}{\partial \bar{x}^{j}} \frac{\partial x^{q}}{\partial \bar{x}^{k}} - \frac{\partial^{2} \bar{x}^{i}}{\partial x^{m} \partial x^{n}} \frac{\partial x^{m}}{\partial \bar{x}^{j}} \frac{\partial x^{n}}{\partial \bar{x}^{k}} \\ \bar{B}^{i}_{jk} = B^{m}_{pq} \frac{\partial \bar{x}^{i}}{\partial x^{m}} \frac{\partial x^{p}}{\partial \bar{x}^{j}} \frac{\partial x^{q}}{\partial \bar{x}^{k}} + \frac{\partial^{2} x^{m}}{\partial \bar{x}^{j} \partial \bar{x}^{k}} \frac{\partial \bar{x}^{i}}{\partial x^{m}} \end{array}}$$

$$\cancel{\frac{d^{2} x^{i}}{ds^{2}} =} \quad f_{1} = \left(\frac{\partial h_{1j}}{\partial x^{k}} - \frac{\partial h_{jk}}{\partial x^{1}} \right) \frac{dx^{j}}{ds} \frac{dx^{k}}{ds}$$

$$f_{2} = \left(\frac{\partial h_{2j}}{\partial x^{k}} - \frac{\partial h_{jk}}{\partial x^{2}} \right) \frac{dx^{j}}{ds} \frac{dx^{k}}{ds}$$

$$f_{3} = \left(\frac{\partial h_{3j}}{\partial x^{k}} - \frac{\partial h_{jk}}{\partial x^{3}} \right) \frac{dx^{j}}{ds} \frac{dx^{k}}{ds}$$

$$f_{4} = \left(\frac{\partial h_{4j}}{\partial x^{k}} - \frac{\partial h_{jk}}{\partial x^{4}} \right) \frac{dx^{j}}{ds} \frac{dx^{k}}{ds}$$

$$f_i = \frac{1}{2}\left(\frac{\partial h_{ij}}{\partial x^k} + \frac{\partial h_{ik}}{\partial x^j} - 2\frac{\partial h_{jk}}{\partial x^i}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}$$

$$\Delta^{mn} f_m f_n = \frac{1}{4}\Delta^{mn}\left(\frac{\partial h_{mj}}{\partial x^k} + \frac{\partial h_{mk}}{\partial x^j} - 2\frac{\partial h_{jk}}{\partial x^m}\right)\left(\frac{\partial h_{nr}}{\partial x^s} + \frac{\partial h_{ns}}{\partial x^r} - 2\frac{\partial h_{rs}}{\partial x^n}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}\frac{dx^r}{ds}\frac{dx^s}{ds}$$

$\cancel{\Delta^{mn} f_m f_n = \frac{1}{4}}$

$$f_i = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}$$

$$\Delta^{mn} f_m f_n = \Delta^{mn}\left(\frac{\partial h_{mj}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^m}\right)\left(\frac{\partial h_{nr}}{\partial x^s} - \frac{\partial h_{rs}}{\partial x^n}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}\frac{dx^r}{ds}\frac{dx^s}{ds}$$

$$\Delta^{mn} f_m f_n = \left[\Delta^{mn}\frac{\partial h_{mj}}{\partial x^k}\frac{\partial h_{nr}}{\partial x^s} - 2\Delta^{mn}\frac{\partial h_{mj}}{\partial x^k}\frac{\partial h_{rs}}{\partial x^n} + \Delta^{mn}\frac{\partial h_{jk}}{\partial x^m}\frac{\partial h_{rs}}{\partial x^n}\right]\frac{dx^j}{ds}\frac{dx^k}{ds}\frac{dx^r}{ds}\frac{dx^s}{ds}$$

[Curvature]

$$\boxed{f_i = B_{ijk} v^j v^k}$$

B_{122}

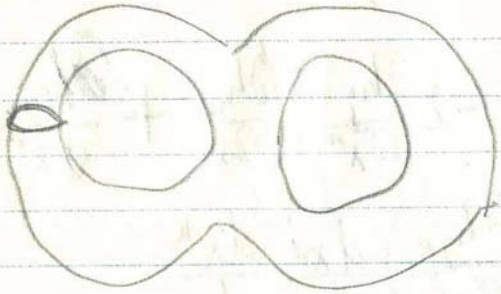
B_{133}

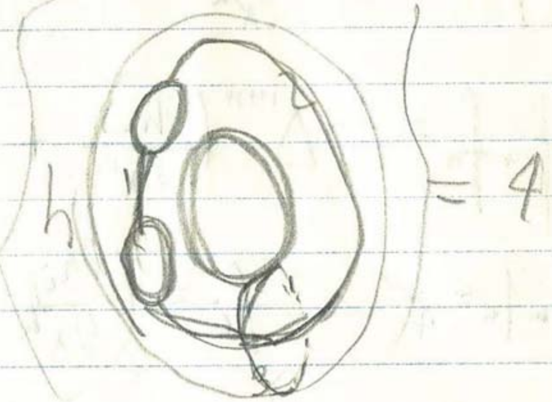
B_{144}

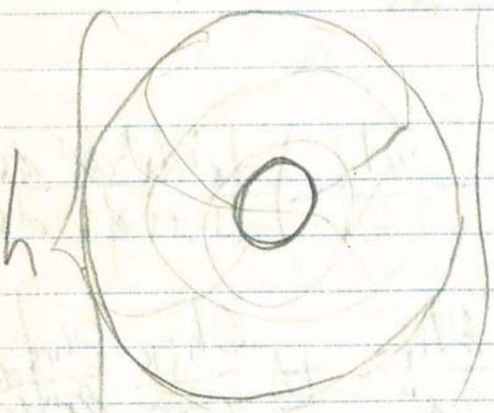
$\cancel{f_i = B_{1jk}\left(1 - (v^2)^2 - (v^3)^2 - (v^4)^2\right] + B_{2jk}}$

$f_1 = B_{1..}$

$$f_1 = B_{111}(v^1)^2 + B_{122}(v^2)^2 + B_{133}(v^3)^2 + B_{144}(v^4)^2$$

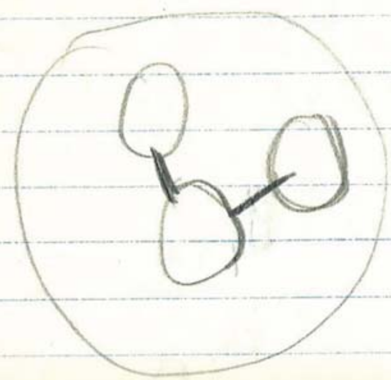


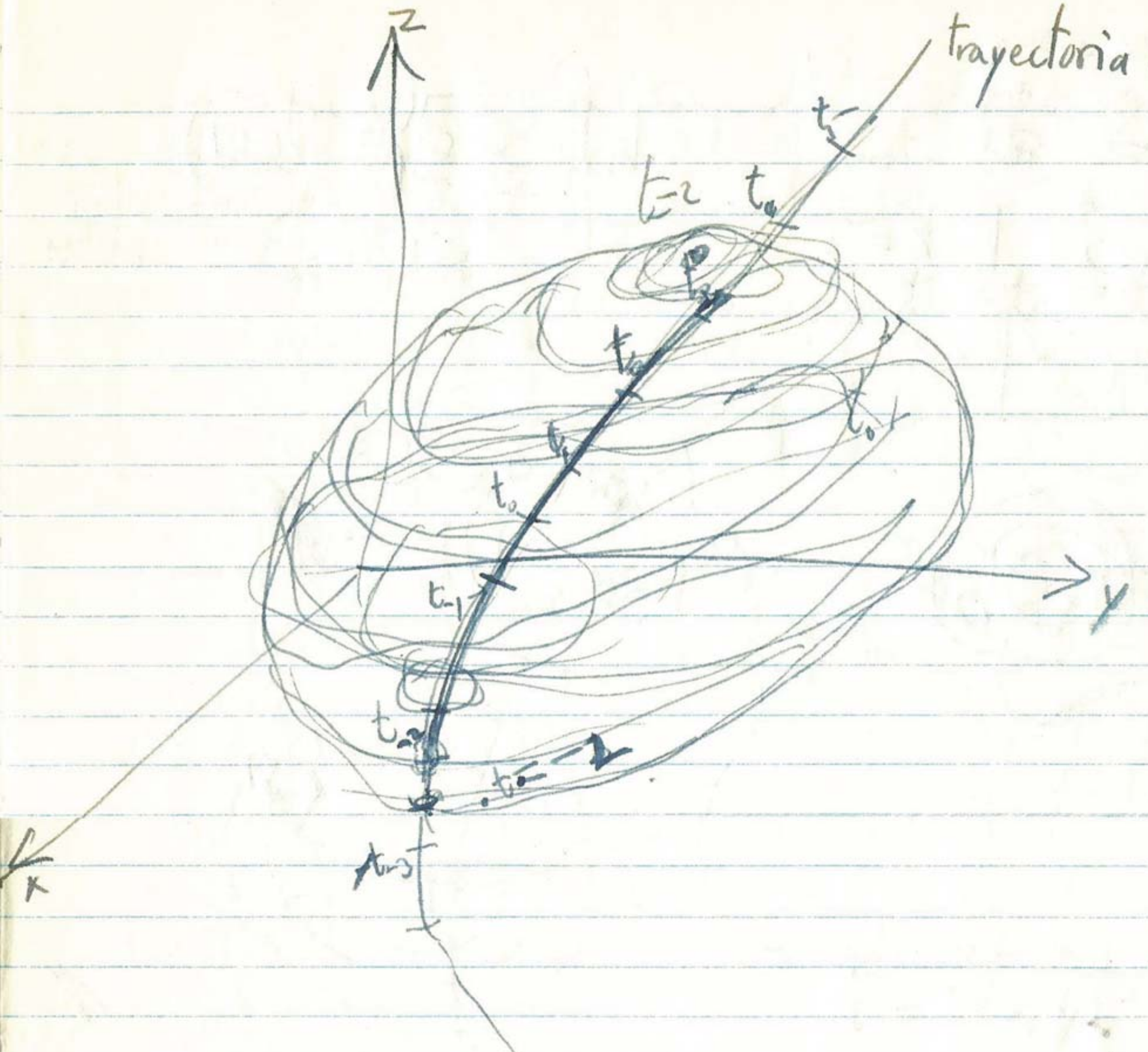




$h\left\{\begin{matrix}\text{esfera con}\\ \text{una perf}\end{matrix}\right\} = 1$

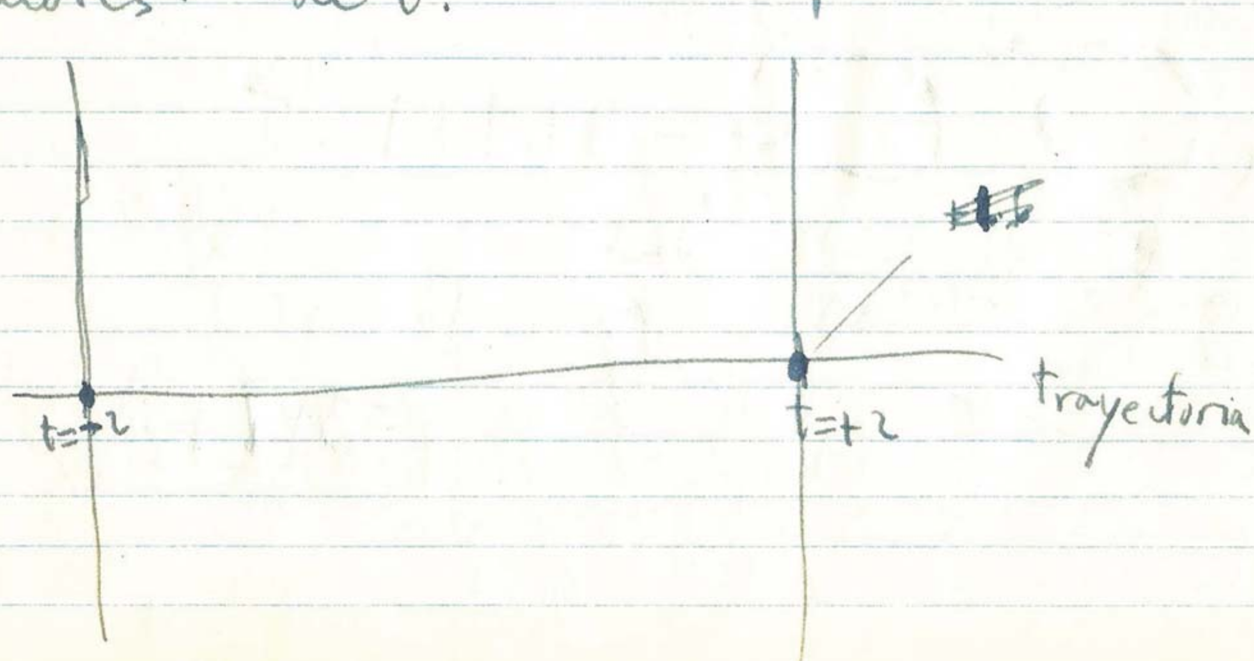
$h\left\{\begin{matrix}\text{esfera con}\\ \text{2 perf}\end{matrix}\right\} = 2$



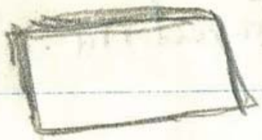


1) Trayectoria ~~acotada~~ parametrizada con valores de t

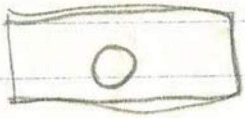
2) Una superficie cerrada "parametrizada con valores de t.



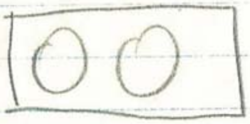
h de um



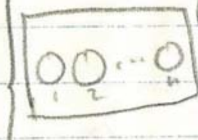
$h=1$

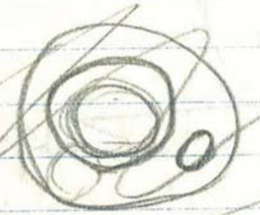


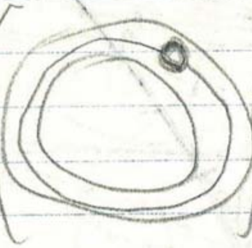
$h=2$

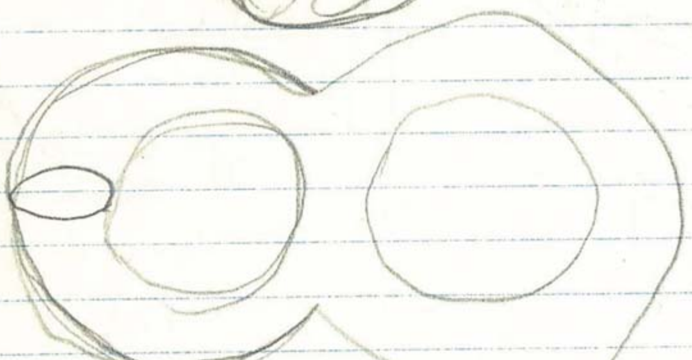


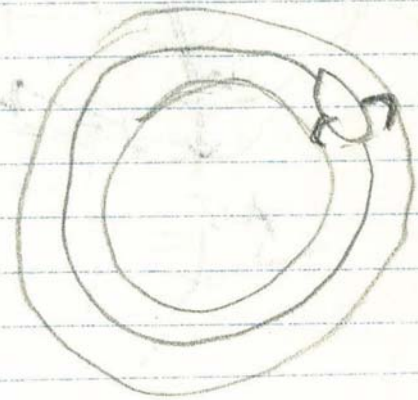
$h=3$

$h\Big\{$  $\Big\} = (n+1)$

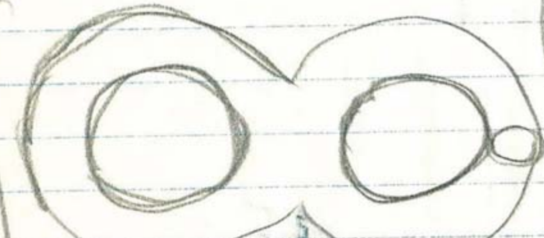
h 

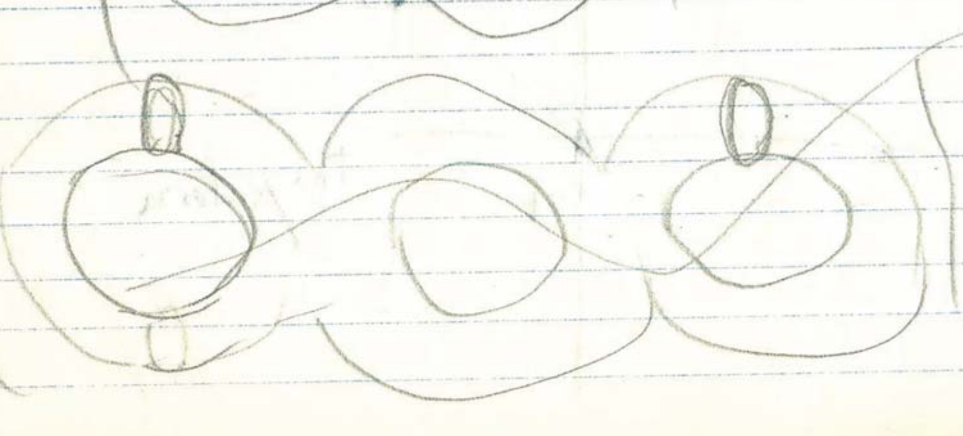
$h\Big\{$  $\Big\} =$ [illegible]





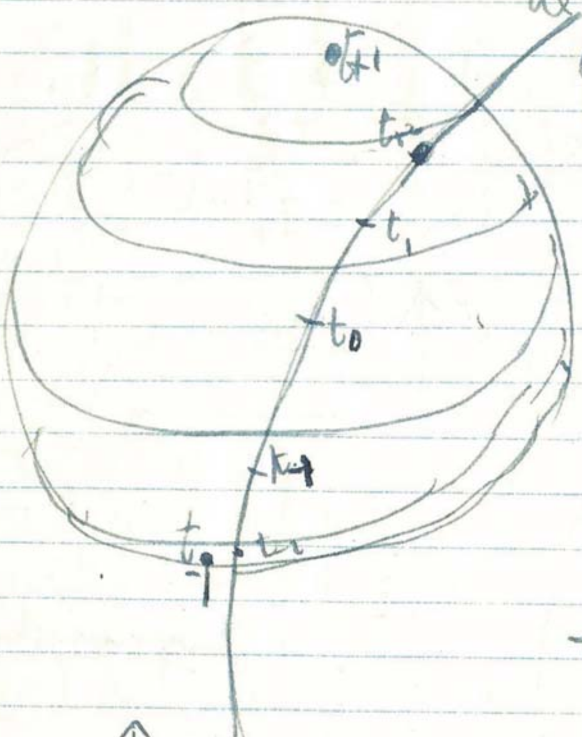
$h\{\text{toro} + n \text{ agujeros}\} = 3 + (n-1)$

$h\Big\{$  $\Big\} = 3+1+1 = 5$

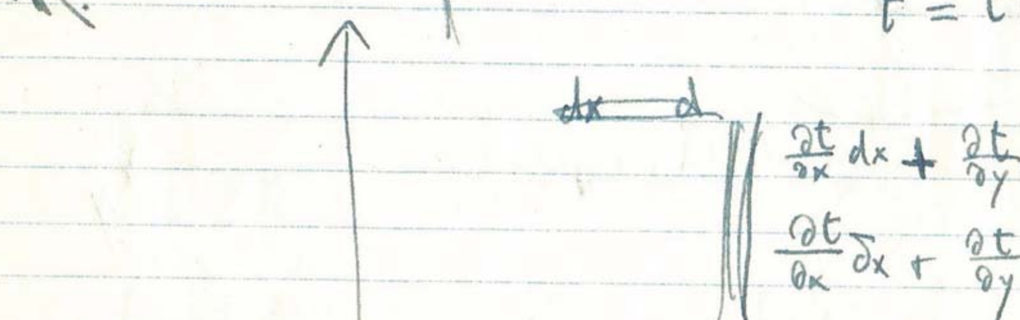
$h\Big\{$  $\Big\} = 3+2+3 = 8$

La superficie es la imagen ~~to~~ topológica de la esfera. Las líneas $t = \text{const.}$ las imágenes topológicas de los paralelos. Sean ~~los~~ polos t_{+1}, t_{-1}

La parte de la línea de universo que queda encerrada está entre t_{-2} y t_{+2}



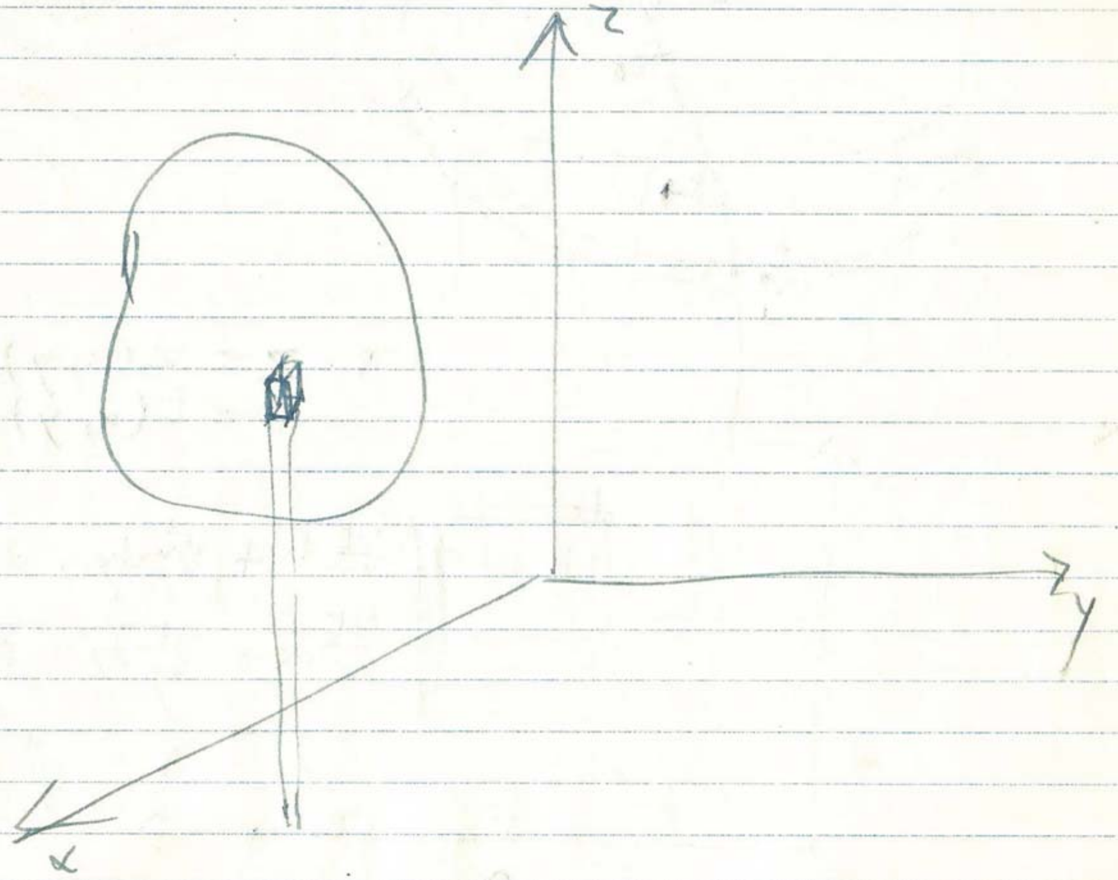
$$z = z(x, y)$$
$$t = t(x, y)$$



~~dx~~ ~~d~~

$$\left\| \begin{matrix} \frac{\partial t}{\partial x} dx + \frac{\partial t}{\partial y} dy & dx & dy & \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \\ \frac{\partial t}{\partial x} \delta x + \frac{\partial t}{\partial y} \delta y & \delta x & \delta y & \frac{\partial z}{\partial x} \delta x + \frac{\partial z}{\partial y} \delta y \end{matrix} \right\|$$

$$\frac{\partial}{\partial z} f(x, y, z, t)$$



z
y
x

Elemento de volumen tridimensional en el espacio tiempo de Minkowski.

$$t = t(x, y, z)$$

$$\cancel{\begin{vmatrix} \frac{\partial t}{\partial x}dx + \frac{\partial t}{\partial y}dy + \frac{\partial t}{\partial z}dz & dx & dy & dz \\ \frac{\partial t}{\partial x}\delta x + \frac{\partial t}{\partial y}\delta y + \frac{\partial t}{\partial z}\delta z & \delta x & \delta y & \delta z \\ \frac{\partial t}{\partial x}Dx + \frac{\partial t}{\partial y}Dy + \frac{\partial t}{\partial z}Dz & Dx & Dy & Dz \end{vmatrix}}$$

~~Tres vectores que~~

Tres cuadrivectores que definen el volumen.

$$\left(\frac{\partial t}{\partial x}dx, \quad dx, \quad 0, \quad 0\right)$$

$$\left(\frac{\partial t}{\partial y}dy, \quad 0, \quad dy, \quad 0\right)$$

$$\left(\frac{\partial t}{\partial z}dz, \quad 0, \quad 0, \quad dz\right)$$

$$\iiint \frac{\partial f}{\partial t}\, dx\, dy\, dz$$

Elemento de volumen. Ecuación de la hipersuperficie $t = t(x, y, z)$

$$\left\| \begin{matrix} \frac{\partial t}{\partial x} dx & dx & 0 & 0 \\ \frac{\partial t}{\partial y} dy & 0 & dy & 0 \\ \frac{\partial t}{\partial z} dz & 0 & 0 & dz \end{matrix} \right\|$$

$$\left[dx\,dy\,dz, -\frac{\partial t}{\partial x} dx\,dy\,dz, -\frac{\partial t}{\partial y} dx\,dy\,dz, -\frac{\partial t}{\partial z} dx\,dy\,dz \right]$$

$$\left[1, -\frac{\partial t}{\partial x}, -\frac{\partial t}{\partial y}, -\frac{\partial t}{\partial z} \right] dx\,dy\,dz$$

Cuadrivector elemento de volumen

$$dV_i = \left[1, -\frac{\partial t}{\partial x}, -\frac{\partial t}{\partial y}, -\frac{\partial t}{\partial z} \right] dx\,dy\,dz$$

Elemento de superficie

Ecuaciones de la superficie

$$t = t(x, y)$$
$$z = z(x, y)$$

$$\left\| \begin{matrix} \frac{\partial t}{\partial x} dx & dx & 0 & \frac{\partial z}{\partial x} dx \\ \frac{\partial t}{\partial y} dy & 0 & dy & \frac{\partial z}{\partial y} dy \end{matrix} \right\|$$

$$dS_{12} = -\frac{\partial z}{\partial x} dx\,dy$$

$$dS_{13} = -\frac{\partial z}{\partial y} dx\,dy$$

$$\partial S_{14} = dx\,dy$$

$$dS_{21} = -dS_{12}$$

$$dS_{23} = \left(\frac{\partial t}{\partial x}\frac{\partial z}{\partial y} - \frac{\partial t}{\partial y}\frac{\partial z}{\partial x} \right) dx\,dy$$

$$dS_{24} = -\frac{\partial t}{\partial x} dx\,dy$$

$$dS_{34} = -\frac{\partial t}{\partial y}\,dx\,dy$$

$$z - z(x,y) = 0$$

$$\nabla f = \left(-\frac{\partial z}{\partial x},\ -\frac{\partial z}{\partial y},\ 1\right)$$

$$\cancel{dS = (A,\ B,\ C)\,dx\,dy}$$

$$\cancel{\nabla f}\ dS^{\alpha} = \nabla f\,dx\,dy = \left(-\frac{\partial z}{\partial x}dx\,dy,\ -\frac{\partial z}{\partial y}dx\,dy,\ dx\,dy\right)$$

$$\iint dS_{ij}\,U^{j} = \iiint \frac{\partial U^{j}}{\partial x^{i}}\,dV_{j}$$

$$\iint\left(\partial S_{12}U^{2} + \partial S_{13}U^{3} + \partial S_{14}U^{4}\right) = \iiint\left(\frac{\partial U^{2}}{\partial x} + \frac{\partial U^{3}}{\partial y} + \frac{\partial U^{4}}{\partial z}\right)dx\,dy\,dz$$

$$U^{2} = U^{2}(t, x, y, z)$$

$$\frac{\partial U^{2}}{\partial x} = \frac{\partial U^{2}}{\partial t}\frac{\partial t}{\partial x} + \frac{\partial U^{2}}{\partial x}\cancel{(x,y)}$$

$$\iiint\left[\frac{\partial U^{1}}{\partial t} + \frac{\partial U^{2}}{\partial x}\frac{\partial t}{\partial x} + \frac{\partial U^{3}}{\partial y}\frac{\partial t}{\partial y} + \frac{\partial U^{4}}{\partial z}\frac{\partial t}{\partial z}\right]dx\,dy\,dz$$

$$\left(\frac{\partial U^{1}}{\partial t},\ \frac{\partial U^{2}}{\partial t},\ \frac{\partial U^{3}}{\partial t},\ \frac{\partial U^{4}}{\partial t}\right)$$

$$\left(\frac{\partial U^{2}}{\partial x},\ \frac{\partial U^{3}}{\partial y},\ \frac{\partial U^{4}}{\partial z}\right)$$

$$\frac{\partial u^2}{\partial x}+\frac{\partial u^3}{\partial y}+\frac{\partial u^4}{\partial z} \quad \frac{\partial u^2}{\partial t} \quad \frac{\partial u^3}{\partial t} \quad \frac{\partial u^4}{\partial t}$$

$$\cancel{\frac{\partial u^3}{\partial y}+\frac{\partial u^4}{\partial z}+\frac{\partial u^1}{\partial t} \quad \partial u^2 \quad \partial u^3 \quad \partial u^4}$$

$$\left|\begin{array}{c|c|c|c}
\frac{\partial u^2}{\partial x}+\frac{\partial u^3}{\partial y}+\frac{\partial u^4}{\partial z} & \frac{\partial u^2}{\partial t} & \frac{\partial u^3}{\partial t} & \frac{\partial u^4}{\partial t} \\
\hline
\frac{\partial u^1}{\partial x} & \frac{\partial u^1}{\partial t}+\frac{\partial u^3}{\partial y}+\frac{\partial u^4}{\partial z} & \frac{\partial u^3}{\partial x} & \frac{\partial u^4}{\partial x} \\
\hline
\frac{\partial u^1}{\partial y} & \frac{\partial u^2}{\partial y} & \frac{\partial u^1}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial u^4}{\partial z} & \frac{\partial u^4}{\partial y} \\
\hline
\frac{\partial u^1}{\partial z} & \frac{\partial u^2}{\partial z} & \frac{\partial u^3}{\partial z} & \frac{\partial u^1}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial u^3}{\partial y}
\end{array}\right|$$

$$\left(\operatorname{div}\bar{u}-\nabla u^1,\ \operatorname{div}\bar{u}-\nabla u^2,\ \operatorname{div}\bar{u}-\nabla u^3,\ \operatorname{div}\vec{u}-\nabla u^4\right)$$

$$\cancel{\mathrm{Rot}} \quad \cancel{P(n) = n}$$

$$\boxed{P(n+1) = n^{P(n)}}$$

$$\int u\,dv = uv - \int v\,du$$

$$d(u^v) = u^v \ln u \, dv + v u^{v-1} du$$

$$\int u^v \ln u \, dv = u^v - \int v u^{v-1} du$$

$$\int v u^{v-1} du = u^v - \int u^v \ln u \, dv$$

$$u = e^x$$

$$\int v e^{x(v-1)} e^x dx = e^{vx} - \int x e^{vx} dv \qquad e^v = dw$$

$$\int v e^{vx} dx = e^{xv} - \int x e^{vx} dv$$

$$v e^{vx} = e^{vx}\left(x \frac{dv}{dx} + v\right) - x e^{vx} \frac{dv}{dx}$$

$$\int \ln w \cdot w^x dx = w^x - \int x w^x d$$

$$\int e^x e^{xe^x} dx = e^{xe^x} - \int x e^{xe^x} e^x dx$$

$$\int_0^\infty e^{x^n} dx = \int_0^\infty \left[e^{-x^{n-1}}\right]^x dx$$

$$\int$$

$$n^{P(n)} = P(n+1)$$

$$P(n)\ln n = \ln P(n+1)$$

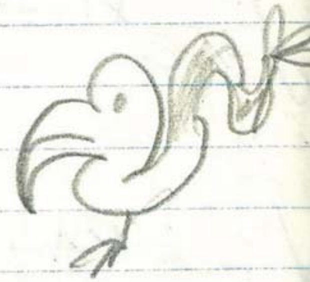
$$\int_0^\infty e^{-x^n} f(x)\,dx = \int_0^\infty e^{-x^n}\, n(x^{n-1}) \frac{f(x)}{n x^{n-1}}\,dx$$

$$= e^{-x^n} \frac{f(x)}{n x^{n-1}}\Bigg| - \int e^{-x^n} \frac{d}{dx}\frac{f(x)}{n x^{n-1}}\,dx$$

$$\cancel{f(x)\,dx}$$

$$\lim_{\alpha \to 0} \alpha^{\alpha} = \lambda$$

$$y'\,dx \quad dx$$

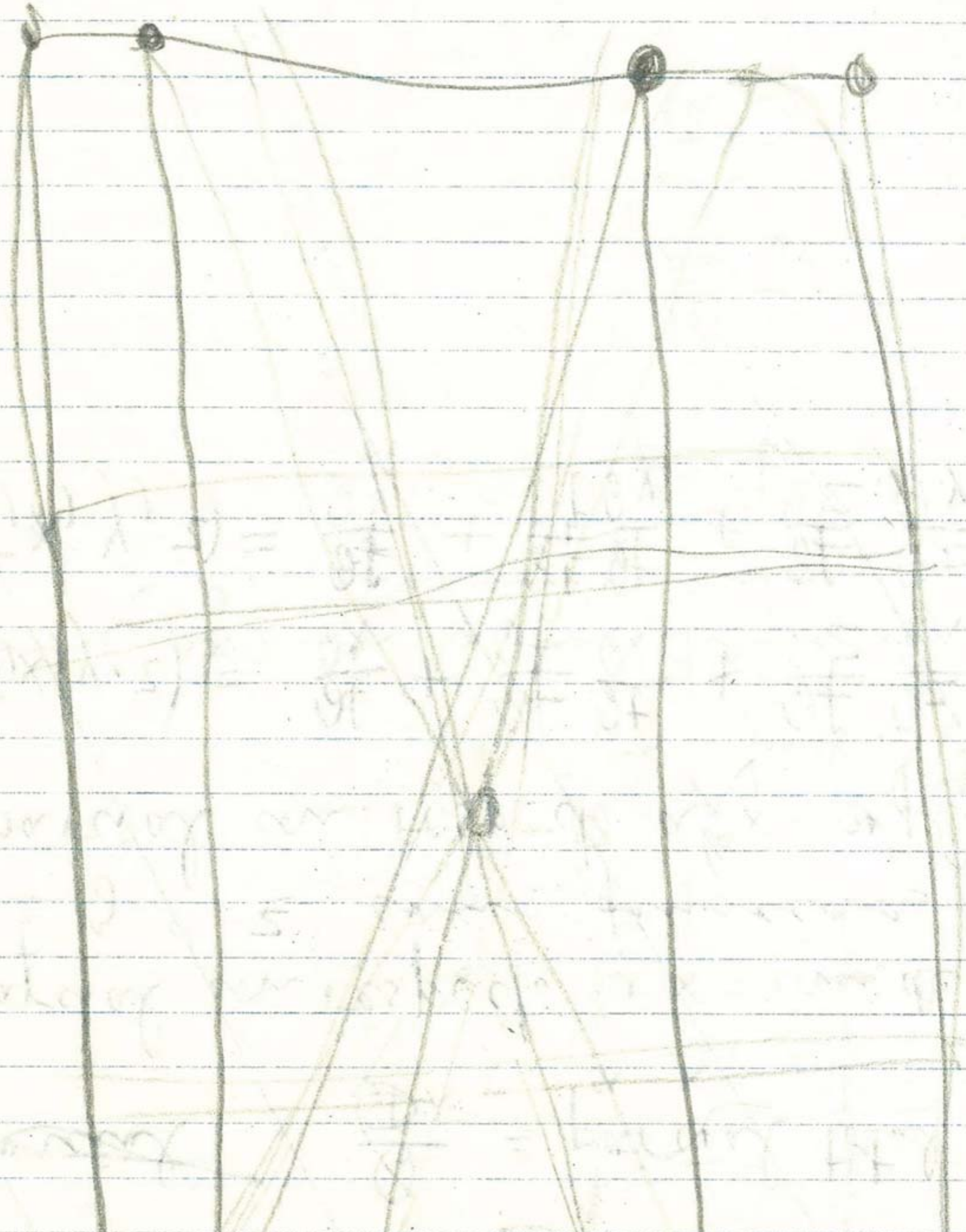


$$\alpha^{\alpha} = f(\alpha)$$

$$\ln f = \alpha \ln \alpha$$

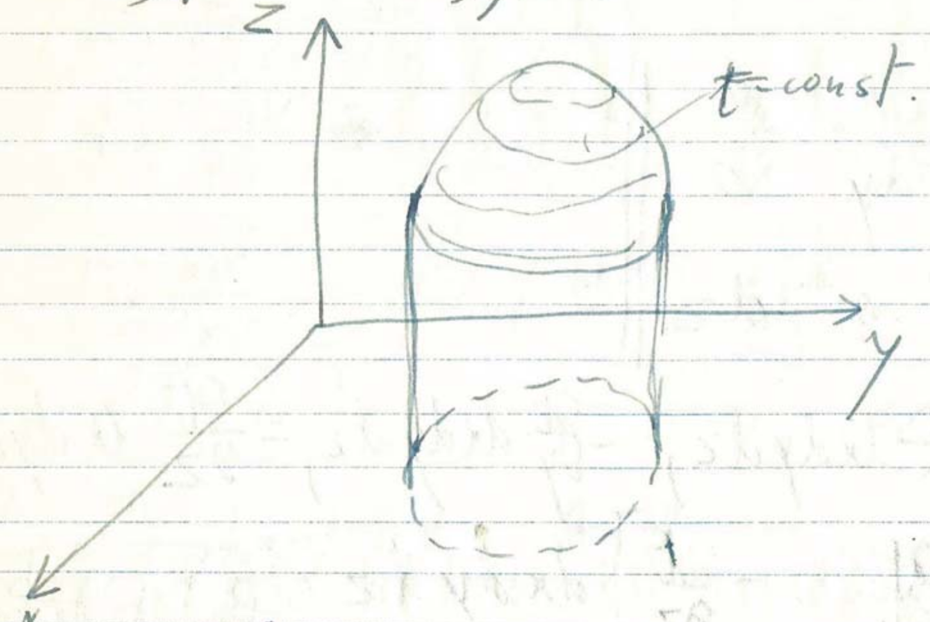
$$\ln f = \frac{\ln \alpha}{\left(\frac{1}{\alpha}\right)}$$

$$\lim_{\alpha \to 0} \ln f = \lim_{\alpha \to 0} \frac{\left(\frac{1}{\alpha}\right)}{\left(-\frac{1}{\alpha^2}\right)} = \lim_{\alpha \to 0} \alpha = 0$$



Volumen en el espacio-tiempo.

$\cancel{t(x,y)}$ $t(x,y,z)$



$f(t,x,y,z)$

$\frac{\partial}{Dx} = \Big\{$ parcial en x considerando a t como función de x

$$\frac{\partial f}{Dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial t}\frac{\partial t}{\partial x}$$

$$\frac{\partial v^2}{Dx} = \frac{\partial v^2}{\partial x} + \frac{\partial v^2}{\partial t}\frac{\partial t}{\partial x}$$

$$\frac{\partial v^3}{\partial y} = \frac{\partial v^3}{\partial y} + \frac{\partial v^3}{\partial t}\frac{\partial t}{\partial y}$$

$$\frac{\partial v^4}{\partial z} = \frac{\partial v^4}{\partial y} + \frac{\partial v^4}{\partial t}\frac{\partial t}{\partial z}$$

Elemento de volumen en el espacio-tiempo.

$$t = t(x, y, z)$$

$$\left\| \begin{matrix} \frac{\partial t}{\partial x} dx & dx & 0 & 0 \\ \frac{\partial t}{\partial y} dy & 0 & dy & 0 \\ \frac{\partial t}{\partial z} dz & 0 & 0 & dz \end{matrix} \right\|$$

$$dV_i = \left[dx\,dy\,dz, -\frac{\partial t}{\partial x} dx\,dy\,dz, -\frac{\partial t}{\partial y} dx\,dy\,dz, -\frac{\partial t}{\partial z} dx\,dy\,dz \right]$$

$$dV_i = \left[1, -\frac{\partial t}{\partial x}, -\frac{\partial t}{\partial y}, -\frac{\partial t}{\partial z} \right] dx\,dy\,dz$$

Elemento de Hipervolumen

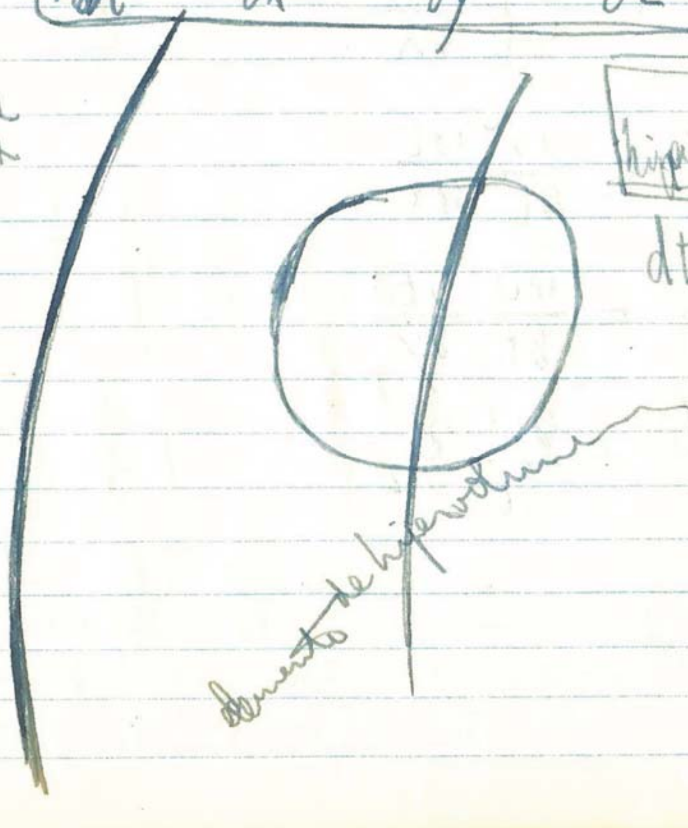
$$v^i = \left[v^1, v^2, v^3, v^4 \right]$$

$$\cancel{v^i dV_i = \Sigma v^i t} \qquad \boxed{\frac{\partial v^1}{\partial t} - \frac{\partial v^2}{\partial x} - \frac{\partial v^3}{\partial y} - \frac{\partial v^4}{\partial z}}$$

$\boxed{\text{hipervolumen}}$

$dt\,dx\,dy\,dz$

$\epsilon_{ijkl}\, dx^i\, \delta x^j\, Dx^k\, \mathcal{D}x^l$

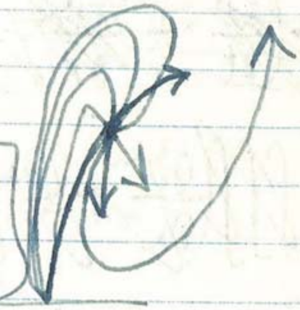


(v^1, v^2, v^3, v^4)

$$\frac{\partial v^1}{\partial t} - \frac{\partial v^2}{\partial t}\frac{\partial t}{\partial x} - \frac{\partial v^3}{\partial t}\frac{\partial t}{\partial y} - \frac{\partial v^4}{\partial t}\frac{\partial t}{\partial z} - \frac{\partial v^2}{\partial x} - \frac{\partial v^3}{\partial y} - \frac{\partial v^4}{\partial z}$$

$$\left[\frac{\partial v^1}{\partial t}, \frac{\partial v^2}{\partial t}, \frac{\partial v^3}{\partial t}, \frac{\partial v^4}{\partial t}\right]$$

$$dV_i \frac{\partial v^i}{\partial t} = dV_1\left[\frac{\partial v^2}{\partial x} + \frac{\partial v^3}{\partial y} + \frac{\partial v^4}{\partial z}\right]$$



$$dV_i \frac{\partial v^i}{\partial x} - dV_2\left(\frac{\partial v^1}{\partial t} + \frac{\partial v^3}{\partial y} + \frac{\partial v^4}{\partial z}\right)$$

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$$\cancel{\operatorname{div} v^i = \frac{\partial v^1}{\partial t} + \frac{\partial v^2}{\partial t}\frac{\partial t}{\partial x} + \frac{\partial v^3}{\partial t}\frac{\partial t}{\partial y} + \frac{\partial v^4}{\partial t}\frac{\partial t}{\partial z} + \frac{\partial v^2}{\partial x} + \frac{\partial v^3}{\partial y} + \frac{\partial v^4}{\partial z}}$$

$$\frac{\partial v^1}{\partial t} + \frac{\partial v^2}{\partial x} + \frac{\partial v^3}{\partial y} + \frac{\partial U^4}{\partial z} = \operatorname{div} v^i$$

$$\frac{\partial v^1}{\partial t} + \frac{\partial v^2}{\partial t}\frac{\partial t}{\partial x} + \frac{\partial v^3}{\partial t}\frac{\partial t}{\partial y} + \frac{\partial v^4}{\partial t}\frac{\partial t}{\partial z} + \frac{\partial v^2}{\partial x} + \frac{\partial v^3}{\partial y} + \frac{\partial v^4}{\partial z} = \operatorname{div} v^i$$

$$(\operatorname{div})_3\, v^i = \frac{\partial v^2}{\partial t}\frac{\partial t}{\partial x} + \frac{\partial v^3}{\partial t}\frac{\partial t}{\partial y} + \frac{\partial v^4}{\partial t}\frac{\partial t}{\partial z} + \frac{\partial v^2}{\partial x} + \frac{\partial v^3}{\partial y} + \frac{\partial v^4}{\partial z}$$

$$\cancel{\frac{\partial v^2}{\partial t}\frac{\partial t}{\partial x} + \frac{\partial v^2}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial v^2}{\partial z}\frac{\partial z}{\partial x} = \frac{\partial v^2}{\partial x}}$$

$$v^2(t, x, y, z) = \cancel{\frac{\partial v^2}{\partial t}}\ \frac{\partial v^2}{\partial t}dt + \frac{\partial v^2}{\partial x}dx + \frac{\partial v^2}{\partial y}dy + \frac{\partial v^2}{\partial z}dz$$

$$z'' = 0$$

$$t't'' - x'x'' - y'y'' - z'z'' = 0$$

$$t't'' = -\frac{m x x'}{r^3} - \frac{2m x x'}{r^3}(x'^2+y'^2) + \frac{m x'^2 r'}{r^2}$$

$$-\frac{m y y'}{r^3} - \frac{2m y y'}{r^3}(x'^2+y'^2) + \frac{m y'^2 r'}{r^2}$$

$$t't'' =$$

$$\frac{d^2t}{ds^2} = \left(\frac{\partial h_{1j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial t}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}$$

$$t'' = \frac{\partial h_{11}}{\partial x^k}\frac{dt}{ds}\frac{dx^k}{ds}$$

$$t'' = m\left[\frac{\partial}{\partial x}\frac{1}{r}\frac{dx}{ds} + \frac{\partial}{\partial y}\frac{1}{r}\frac{dy}{ds} + \frac{\partial}{\partial z}\frac{1}{r}\frac{dz}{ds}\right]\frac{dt}{ds}$$

$$\cancel{t' = m\left[-\frac{x}{r^2} - \frac{\quad}{r^2}\right.}$$

$$t'' = m\left[-\frac{xx'}{r^3} - \frac{yy'}{r^3} - \frac{zz'}{r^3}\right]t'$$

$$\cancel{t'' = -\frac{mxx't'}{r^3} - myy'}$$

$$t'' = -\frac{mt'}{r^3}\left[xx' + yy'\right] + \cancel{zz'}$$

$$x'' = -\frac{mx}{r^3} - \frac{2mx}{r^3}(x'^2 + y'^2) + \frac{mx'r'}{r^2}$$

$$y'' = -\frac{my}{r^3} - \frac{2my}{r^3}(x'^2 + y'^2) + \frac{my'r'}{r^2}$$

$$t't'' - x'x'' - y'y'' =$$

$$= -\frac{mt'^2}{r^3}\left[xx' + yy'\right] + \frac{mxx'}{r^3} + \frac{2mxx'}{r^3}(x'^2 + y'^2) - \frac{mx'^2r'}{r^2}$$

$$+ \frac{myy'}{r^3} + \frac{2myy'}{r^3}(x'^2 + y'^2) - \frac{my'^2r'}{r^2} =$$

$$-t'^2 + x'^2 + y'^2 = -1$$

$$(xx' + yy')\frac{m}{r^3}\left[-t'^2 + 1 + 2x'^2 + 2y'^2\right] - \frac{mr'}{r^2}\left[x'^2 + y'^2\right]$$

$$\frac{m(xx' + yy')}{r^3}\left[x'^2 + y'^2\right] - \frac{mr'}{r^2}(x'^2 + y'^2)$$

$$\cancel{\left[\frac{m(xx' + yy')}{r^3} - \frac{m(xx' + yy')}{r^3}\right](x'^2 + y'^2)} = 0$$

$$t'' = -\frac{mt'}{r^3}(xx' + yy') = -\frac{mt'}{r^3} rr'$$

$$x'' = -\frac{mx}{r^3} - \frac{2mx}{r^3}(x'^2 + y'^2) + \frac{mx' rr'}{r^3}$$

$$y'' = -\frac{my}{r^3} - \frac{2my}{r^3}(x'^2 + y'^2) + \frac{my' rr'}{r^3}$$

$$\frac{r^6 \kappa^2}{m^2} = (1 - x'^2 - y'^2)(x^2 x'^2 + 2xyx'y' + y^2 y'^2)$$
$$+ x^2 + y^2 + 4x^2(x'^4 + 2x'^2 y'^2 + y'^4)$$
$$+ x'^2 ($$

$$\frac{r^6 \kappa^2}{m^2} = t'^2 r^2 r'^2$$
$$- (x^2 + y^2) + 4(x^2 + y^2)(x'^2 + y'^2)^2 - (x'^2 + y'^2) r^2 r'^2$$
$$- 4(x^2 + y^2)(x'^2 + y'^2) + 2(xx' + yy') rr'$$
$$+ 4(xx' + yy') rr' (x'^2 + y'^2)$$

$$\frac{r^6 \kappa^2}{m^2} = r^2 r'^2 - r^2 + 4r^2(x'^2 + y'^2)^2$$
$$+ 4r^2(x'^2 + y'^2)[x'^2 + y'^2 - 1]$$
$$+ \cancel{2rr'}(x \cdot + 2r^2 r'^2 [2(x'^2 + y'^2) + 1]$$

$$t'' = -\frac{mt'}{r^3} r r'$$

$$x'' = -\frac{mx}{r^3} - \frac{2mx}{r^3}(x'^2+y'^2) + \frac{m x' r r'}{r^3}$$

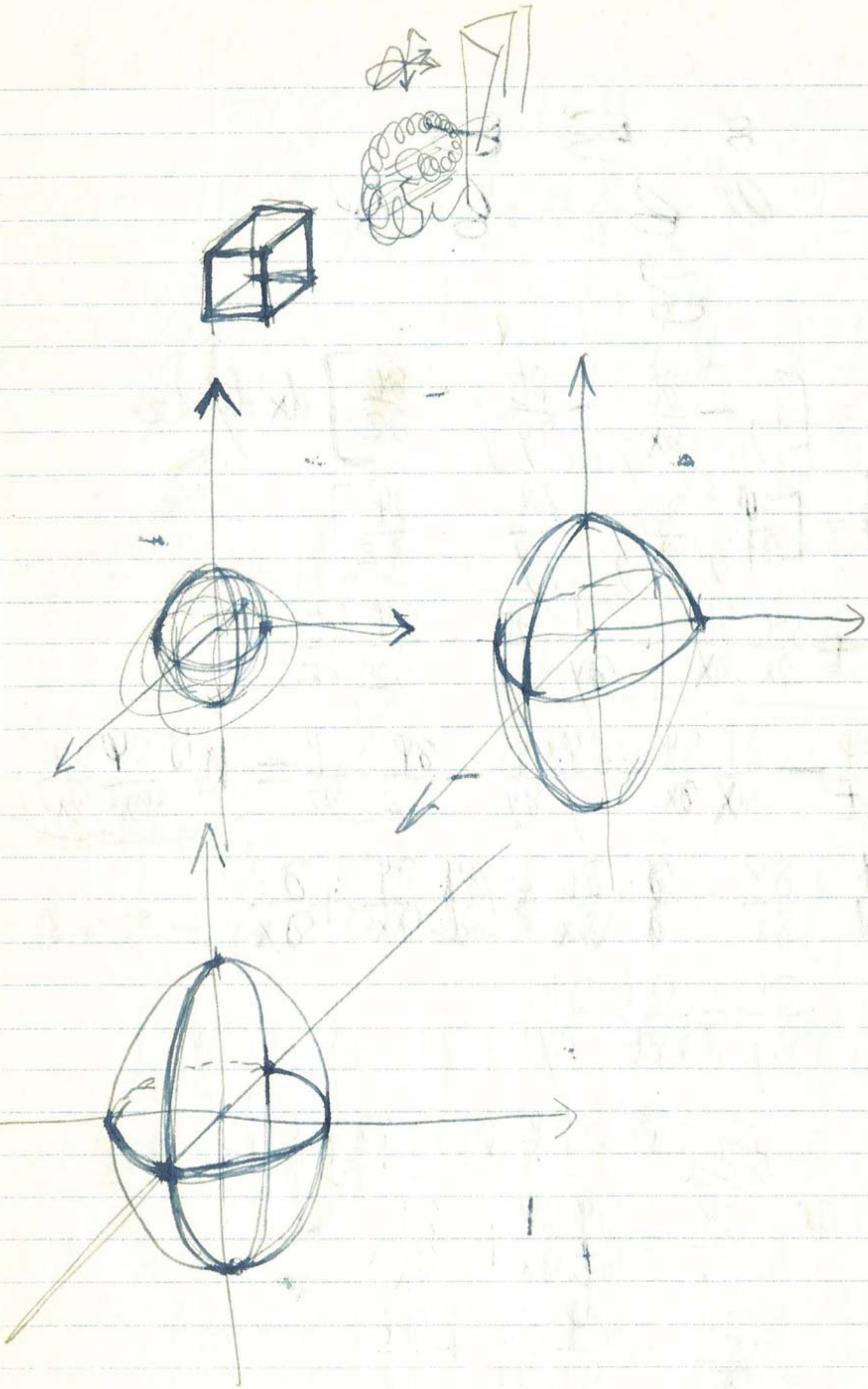
$$y'' = -\frac{my}{r^3} - \frac{2my}{r^3}(x'^2+y'^2) + \frac{m y' r r'}{r^3}$$

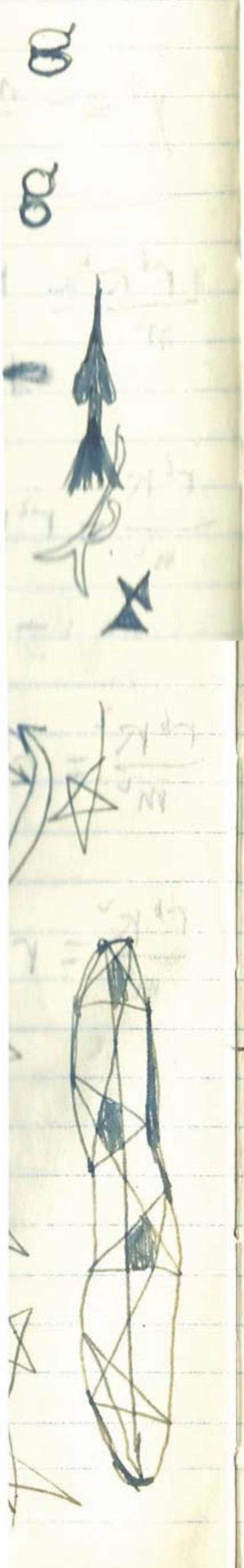
$$\frac{r^6 k^2}{m^2} = r^2 r'^2 t'^2 - r^2 - 4r^2(x'^2+y'^2)^2 - (x'^2+y'^2) r^2 r'^2$$
$$- 4r^2(x'^2+y'^2) + 2r^2 r'^2 + 4r^2 r'^2 (x'^2+y'^2)$$

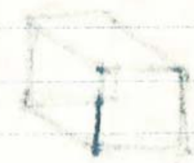
$$\frac{r^6 k^2}{m^2} = r^2 r'^2 \left[\underbrace{t'^2 - x'^2 - y'^2} + 2 + 4x'^2 + 4y'^2\right]$$
$$- r^2 - 4r^2(x'^2+y'^2)(1 + x'^2 + y'^2)$$

$$\frac{r^6 k^2}{m^2} = r^2 r'^2 \left[3 + 4x'^2 + 4y'^2\right] - r^2 - 4r^2(x'^2+y'^2)t'^2$$

$$\frac{r^6 k^2}{m^2} = r^2 r'^2 \left[3 + 4t'^2 - 4\right] - r^2 - 4r^2 t'^2 (t'^2 - 1)$$







$$dV_i = \left[1, -\frac{\partial t}{\partial x}, -\frac{\partial t}{\partial y}, -\frac{\partial t}{\partial z}\right] dx\,dy\,dz$$

$$\frac{\partial \psi}{\partial x^i} = \left[\frac{\partial \varphi}{\partial t}, \frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}, \frac{\partial \varphi}{\partial z}\right]$$

$$\frac{\partial \varphi}{\partial t} - \frac{\partial \varphi}{\partial x}\frac{\partial t}{\partial x} - \frac{\partial \varphi}{\partial y}\frac{\partial t}{\partial y} - \frac{\partial \varphi}{\partial z}\frac{\partial t}{\partial z}$$

$$\frac{\partial \varphi}{\partial t}\frac{\partial \psi}{\partial t} - \frac{\partial \varphi}{\partial x}\frac{\partial \psi}{\partial x} - \frac{\partial \varphi}{\partial y}\frac{\partial \psi}{\partial y} - \frac{\partial \varphi}{\partial z}\frac{\partial \psi}{\partial z} = \Delta^{ij}\frac{\partial \varphi}{\partial x^i}\frac{\partial \psi}{\partial x^j}$$

$$\cancel{\frac{\partial \varphi}{\partial x}} \quad \frac{\delta \varphi}{\delta x} = \frac{\partial \varphi}{\partial t}\frac{\partial t}{\delta x} \cancel{+ \frac{\partial \varphi}{\partial t}\frac{\partial \psi}{\partial x}} + \frac{\partial \varphi}{\partial x}$$

$$\frac{\delta \varphi}{\delta y} = \frac{\partial \varphi}{\partial t}\frac{\partial \psi}{\partial y}$$

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$$\frac{\delta \varphi}{\delta x} = \frac{\partial \varphi}{\partial t}\frac{\partial \psi}{\partial x} + \frac{\partial \varphi}{\partial x}$$

$$\frac{\delta \varphi}{\delta y} = \frac{\partial \varphi}{\partial t}\frac{\partial \psi}{\partial y} + \frac{\partial \varphi}{\partial y}$$

$$\frac{\delta \varphi}{\delta z} = \frac{\partial \varphi}{\partial t}\frac{\partial \psi}{\partial z} + \frac{\partial \varphi}{\partial z}$$

$$\delta iv\,\varphi = \frac{\partial\varphi}{\partial t}\left[\frac{\partial\psi}{\partial x}+\frac{\partial\psi}{\partial y}+\frac{\partial\psi}{\partial z}\right]+\left[\frac{\partial\varphi}{\partial x}+\frac{\partial\varphi}{\partial y}+\frac{\partial\varphi}{\partial z}\right]$$

$$\delta iv\,\varphi = \frac{\partial\varphi}{\partial t}\left[\frac{\partial t}{\partial x}+\frac{\partial t}{\partial y}+\frac{\partial t}{\partial z}\right]+\frac{\partial\varphi}{\partial t}\frac{\partial t}{\partial t}-\left(\frac{\partial\varphi}{\partial t}+\frac{\partial\varphi}{\partial x}-\frac{\partial\varphi}{\partial y}-\frac{\partial\varphi}{\partial z}\right)$$

$$\delta iv\,\varphi = \underbrace{\frac{\partial\varphi}{\partial t}\left[\frac{\partial t}{\partial t}+\frac{\partial t}{\partial x}+\frac{\partial t}{\partial y}+\frac{\partial t}{\partial z}\right]}-\left[\frac{\partial\varphi}{\partial t}-\frac{\partial\varphi}{\partial x}-\frac{\partial\varphi}{\partial y}-\frac{\partial\varphi}{\partial z}\right)$$

$$\frac{\delta v^2}{\partial x} = \frac{\partial v^2}{\partial t}\frac{\partial t}{\partial x}+\frac{\partial v^2}{\partial x}$$

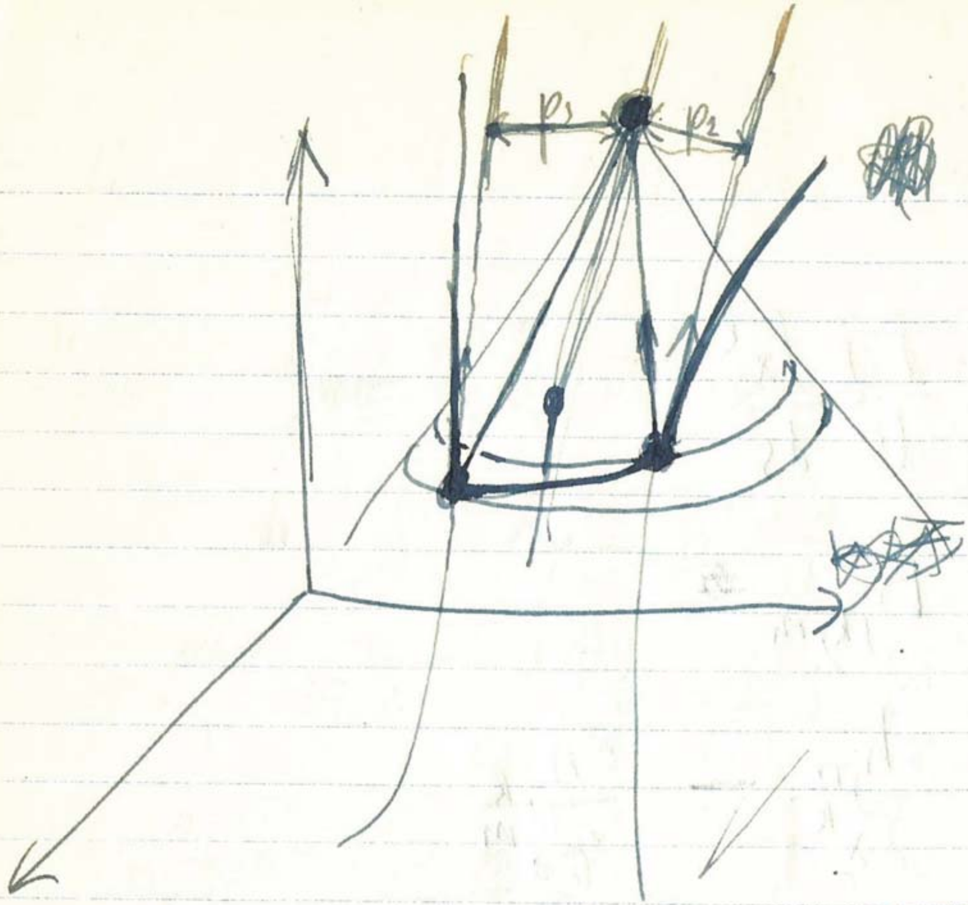
$$\frac{\delta v^3}{\partial y} = \frac{\partial v^3}{\partial t}\frac{\partial t}{\partial y}+\frac{\partial v^3}{\partial y}$$

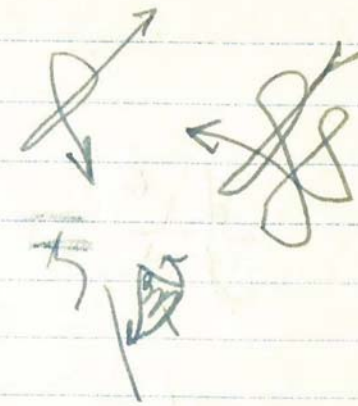
$$\frac{\delta v^4}{\partial z} = \frac{\partial v^4}{\partial t}\frac{\partial t}{\partial y}+\frac{\partial v^4}{\partial z}$$

$$\delta iv\,\vec{v} = \dot{\vec{v}}\cdot\nabla t + div\,\vec{v}$$

$$\iiint \underline{\vec{v}\cdot\nabla t}\, dV = \iiint div(\vec{v}\,t)\,dV - \iiint t\,div\,\vec{v}\,dV$$

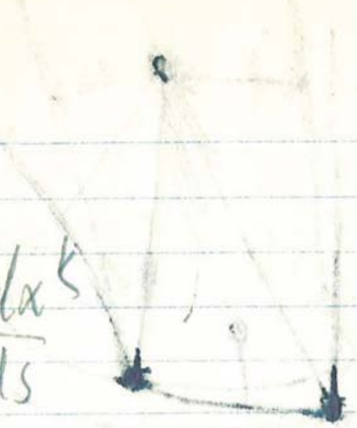
$$div\,\vec{v}\cdot\nabla t \qquad div(\vec{v}\cdot t) = \vec{v}\cdot\nabla t + \underline{t\,div\,\vec{v}}$$





$$\frac{2\underline{V}_i\underline{V}_j - \Delta_{ij}}{|p_1|} + \frac{2\underline{V}_i\underline{V}_j - \Delta_{ij}}{|p_2|}$$

$$\cancel{\frac{2\underline{v}_i\underline{v}_j - \Delta_{ij}}{|p|} =}$$



$$\frac{d^2x^i}{ds^2} = B^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds}$$

$$B^i_{jk} = \Delta^{im} B_{jk,m}$$

$$B_{jk,m} = \frac{\partial h_{jm}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^m}$$

$$\frac{d^2x^i}{ds^2} + B^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

$$\frac{d^2x^i}{ds^2} + \Delta^{im} B_{jk,m}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

$$\frac{d^2x^i}{ds^2} + \Delta^{im} B_{kj\,m}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

$$2\frac{d^2x^i}{ds^2} + \Delta^{im}\left[B_{jk,m} + B_{kj,m}\right]\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

$$\frac{d^2x^i}{ds^2} + \Delta^{im}\left(\frac{1}{2}\left\{B_{jk,m} + B_{kj,m}\right\}\right)\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

$$B_{jk,m} = -\frac{\partial h_{jm}}{\partial x^k} + \frac{\partial h_{jk}}{\partial x^m}$$

$$B_{kj,m} = -\frac{\partial h_{km}}{\partial x^j} + \frac{\partial h_{kj}}{\partial x^m}$$

Campo Central

Tensor de curvatura de Riemann-Christoffel

$$R^{\varepsilon}_{\mu\nu\sigma} = \Gamma^{\alpha}_{\mu\sigma}\Gamma^{\varepsilon}_{\alpha\nu} - \Gamma^{\alpha}_{\mu\nu}\Gamma^{\varepsilon}_{\alpha\sigma} + \frac{\partial}{\partial x^{\nu}}\Gamma^{\varepsilon}_{\mu\sigma} - \frac{\partial}{\partial x^{\sigma}}\Gamma^{\varepsilon}_{\mu\nu}$$

$$h_{ij} = \delta_{ij}\frac{m}{r}$$

$$\frac{\partial h_{ij}}{\partial x^{1}} = 0 \qquad \frac{\partial h_{ij}}{\partial x^{\alpha}} = -\delta_{ij}\frac{m}{r^2}\frac{x^{\alpha}}{r}$$

$$\frac{\partial h_{ij}}{\partial x^{\alpha}} = -\delta_{ij}\frac{m x^{\alpha}}{r^3}$$

$$\Gamma_{\alpha\beta,\gamma} = \left[\frac{\partial h_{\alpha\gamma}}{\partial x^{\beta}} - \frac{\partial h_{\alpha\beta}}{\partial x^{\gamma}}\right]$$

$$\Gamma_{\alpha\beta,\gamma} = \left[-\frac{\delta_{\alpha\gamma}\, m x^{\beta}}{r^3} + \frac{\delta_{\alpha\beta}\, m x^{\gamma}}{r^3}\right]$$

$$\boxed{\Gamma_{\alpha\beta,\gamma} = \frac{m}{r^3}\left[\delta_{\alpha\beta}\, x^{\gamma} - \delta_{\alpha\gamma}\, x^{\beta}\right]}$$

$$\Gamma_{\alpha\beta,1} = \left[\frac{\partial h_{\alpha 1}}{\partial x^{\beta}} - \frac{\partial h_{\alpha\beta}}{\partial x^{1}}\right] = 0$$

$$\boxed{\Gamma_{\alpha\beta,1} = 0}$$

$$\Gamma_{1\beta,\gamma} = \left[\frac{\partial h_{1\gamma}}{\partial x^{\beta}} - \frac{\partial h_{1\beta}}{\partial x^{\gamma}}\right] = 0$$

$$\boxed{\Gamma_{1\beta,\gamma} = 0} \qquad \boxed{\Gamma_{\alpha 1,\gamma} = 0}$$

$$\Gamma_{11,\gamma} = \left[\frac{\partial h_{1\gamma}}{\partial x^1} - \frac{\partial h_{11}}{\partial x^\gamma}\right]$$

$$\Gamma_{11,\gamma} = -\frac{\partial}{\partial x^\gamma} h_{11} = \frac{m}{r^3} x^\gamma$$

$$\boxed{\Gamma_{11,\gamma} = \frac{m x^\gamma}{r^3}}$$

$$\Gamma_{1\beta,1} = \frac{\partial h_{11}}{\partial x^\beta} - \frac{\partial h_{1\beta}}{\partial x^1} = -\frac{m x^\beta}{r^3}$$

$$\boxed{\Gamma_{1\beta,1} = -\frac{m x^\beta}{r^3}}$$

$$\boxed{\Gamma_{11,1} = \frac{\partial h_{11}}{\partial x^1} - \frac{\partial h_{11}}{\partial x^1} = 0}$$

$\boxed{\text{Alternativa}}$

$$h_{ij} = \Delta_{ij}\frac{m}{r}$$ Campo central

$$\frac{d^2x^i}{ds^2} + \Delta^{im}\left[\frac{1}{2}\left(\frac{\partial h_{mj}}{\partial x^k} + \frac{\partial h_{mk}}{\partial x^j} - \frac{\partial h_{jk}}{\partial x^m}\right)\right]\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

Campo central

$$B_{\alpha\beta,2} = \frac{1}{2}\left[\frac{\partial h_{2\alpha}}{\partial x^\beta} + \frac{\partial h_{2\beta}}{\partial x^\alpha} - \frac{\partial h_{\alpha\beta}}{\partial x}\right]$$

$$B_{2\beta,2} = \frac{1}{2}\left[\frac{\partial h_{22}}{\partial x^\beta} + \cancel{\frac{\partial h_{2\beta}}{\partial x}} - \cancel{\frac{\partial h_{2\beta}}{\partial x}}\right]$$

$$B_{22,2} = \frac{1}{2}\left[\frac{\partial h_{22}}{\partial x}\right] \qquad B_{23,2} = \frac{1}{2}\left[\frac{\partial h_{22}}{\partial y}\right]$$

$$B_{\alpha\beta,3} = \frac{1}{2}\left[\frac{\partial h_{\alpha 3}}{\partial x^\beta} + \frac{\partial h_{\beta 3}}{\partial x^\alpha} - \frac{\partial h_{\alpha\beta}}{\partial y}\right]$$

$$B_{22,3} = \frac{1}{2}\left[\frac{\partial h_{23}}{\partial x} + \frac{\partial h_{23}}{\partial x} - \frac{\partial h_{22}}{\partial y}\right] = -\frac{1}{2}\frac{\partial h_{22}}{\partial y}$$

Los Pseudosímbolos de Christoffel y el Tensor de Pseudocurvatura en el Campo Central de Birkhoff.

Las ecuaciones del movimiento de una partícula en la teoría de Birkhoff son:

$$\frac{d^2x^i}{ds^2} = \Delta^{im}\left[\frac{\partial h_{jm}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^m}\right]\frac{dx^j}{ds}\frac{dx^k}{ds}.$$

Estas ecuaciones se pueden escribir de los dos modos siguientes:

$$\frac{d^2x^i}{ds^2} + \Delta^{im}\left[\frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{jm}}{\partial x^k}\right]\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

$$\frac{d^2x^i}{ds^2} + \Delta^{im}\left[\frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{km}}{\partial x^j}\right]\frac{dx^j}{ds}\frac{dx^k}{ds} = 0.$$

Sumando miembro a miembro estas dos ecuaciones y dividiendo entre dos, se obtiene:

$$\frac{d^2x^i}{ds^2} + \Delta^{im}\left[\frac{1}{2}\left\{2\frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{jm}}{\partial x^k} - \frac{\partial h_{km}}{\partial x^j}\right\}\right]\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

$$\frac{d^2x}{ds^2} - \frac{m}{r^3}(xx'+yy')\,x' - \frac{1}{2}\frac{mx}{r^3}\left[t'^2 - x'^2 - y'^2\right] = 0$$

$$\frac{d^2x}{ds^2} - \frac{1}{2}\frac{mx}{r^3} - \frac{mx'}{r^3}(xx'+yy') = 0$$

$$\frac{d^2y}{ds^2} - \frac{1}{2}\frac{my}{r^3} - \frac{my'}{r^3}(xx'+yy') = 0$$

Pseudosímbolos de Christoffel y Tensor de Pseudocurvatura en el Campo Central en la Teoría de la Gravitación de Birkhoff.

Las ecuaciones diferenciales de la línea de universo de una partícula exploradora en la teoría de la gravitación de Birkhoff son:

$$(1) \qquad \frac{d^2x^i}{ds^2} = \Delta^{im}\left(\frac{\partial h_{mj}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^m}\right)\frac{dx^j}{ds}\frac{dx^k}{ds} .$$

Las ecuaciones (1) pueden escribirse de los dos modos siguientes:

$$\frac{d^2x^i}{ds^2} + \Delta^{im}\left[\frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{jm}}{\partial x^k}\right]\frac{dx^j}{ds}\frac{dx^k}{ds} = 0,$$

$$\frac{d^2x^i}{ds^2} + \Delta^{im}\left(\frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{km}}{\partial x^j}\right)\frac{dx^j}{ds}\frac{dx^k}{ds} = 0 .$$

Sumando miembro a miembro estas dos ecuaciones se obtiene:

$$2\frac{d^2x^i}{ds^2} + \Delta^{im}\left[2\frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{jm}}{\partial x^k} - \frac{\partial h_{km}}{\partial x^j}\right]\frac{dx^j}{ds}\frac{dx^k}{ds} = 0.$$

Dividiendo entre 2 :

$$\frac{d^2x^i}{ds^2} + \Delta^{im}\left[\frac{1}{2}\left\{2\frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{jm}}{\partial x^k} - \frac{\partial h_{km}}{\partial x^j}\right\}\right] = 0.$$

Convención:	índices griegos	índices latinos
	$\alpha = 2, 3, 4$	$i = 1, 2, 3, 4$
	$\beta = 2, 3, 4$	$j = 1, 2, 3, 4$
	$\gamma = 2, 3, 4$	$k = 1, 2, 3, 4$

Definición: Los pseudosímbolos de Christoffel de primera especie son por definición:

$$B_{jk,m} = \frac{1}{2}\left\{2\frac{\partial h_{jk}}{\partial x^m} - \frac{\partial h_{jm}}{\partial x^k} - \frac{\partial h_{km}}{\partial x^j}\right\}.$$

Los pseudosímbolos de Christoffel de segunda especie son por definición:

$$B_{jk}^{\,i} = \Delta^{im} B_{jk,m}.$$

Pseudosímbolos de Christoffel de Primera Especie

$$h_{ij} = \delta_{ij}\frac{m}{r}.$$

$$\frac{\partial h_{ij}}{\partial x^\alpha} = -\delta_{ij}\frac{m x^\alpha}{r^3} \; ; \quad \frac{\partial h_{ij}}{\partial x^1} = 0 .$$

$$\boxed{B_{11,1} = 0}$$

$$B_{1k,m} = \frac{1}{2}\left(2\frac{\partial h_{1k}}{\partial x^m} - \frac{\partial h_{1m}}{\partial x^k} - \frac{\partial h_{km}}{\partial x^1}\right)$$

$$B_{1k,m} = \frac{1}{2}\left[2\frac{\partial h_{1k}}{\partial x^m} - \frac{\partial h_{1m}}{\partial x^k}\right]$$

$$B_{11,\alpha} = \frac{1}{2}\left[2\frac{\partial h_{11}}{\partial x^\alpha} - \frac{\partial h_{1\alpha}}{\partial x^1}\right]$$

$$B_{11,\alpha} = \frac{1}{2}2\frac{\partial h_{11}}{\partial x^\alpha} = -\frac{mx^\alpha}{r^3}$$

$$\boxed{B_{11,\alpha} = -\frac{mx^\alpha}{r^3}} \qquad \boxed{B_{11,1} = 0}$$

$$B_{1\alpha,1} = \frac{1}{2}\left[2\frac{\partial h_{1\alpha}}{\partial x^1} - \frac{\partial h_{11}}{\partial x^\alpha}\right]$$

$$\boxed{B_{1\alpha,1} = +\frac{1}{2}\frac{mx^\alpha}{r^3}}$$

$$B_{1\alpha,\beta} = \frac{1}{2}\left[2\frac{\partial h_{1\alpha}}{\partial x^\beta} - \frac{\partial h_{1\beta}}{\partial x^\alpha}\right] = 0$$

$$\boxed{B_{1\alpha,\beta} = 0}$$

$$B_{\alpha\beta,1} = \frac{1}{2}\left[2\frac{\partial h_{\alpha\beta}}{\partial x^1} - \frac{\partial h_{\alpha 1}}{\partial x^\beta} - \frac{\partial h_{\beta 1}}{\partial x^\alpha}\right] = 0$$

$$\boxed{B_{\alpha\beta,1} = 0}$$

$$B_{\alpha\beta,\gamma} = \frac{1}{2}\left[2\frac{\partial h_{\alpha\beta}}{\partial x^\gamma} - \frac{\partial h_{\alpha\gamma}}{\partial x^\beta} - \frac{\partial h_{\beta\gamma}}{\partial x^\alpha}\right]$$

~~Si $\alpha \neq \beta$ y $\beta \neq \gamma$ y $\alpha \neq \gamma$~~

Si α, β, γ son una permutación de 2, 3, 4 entonces $\underline{B_{\alpha\beta,\gamma} = 0}$

$$B_{\alpha\alpha,\gamma} = \frac{1}{2}\left[2\frac{\partial h_{\alpha\alpha}}{\partial x^\gamma} - \frac{\partial h_{\alpha\gamma}}{\partial x^\beta} - \frac{\partial h_{\alpha\gamma}}{\partial x^\alpha}\right]$$

$\alpha \neq \gamma$

$$B_{\alpha\alpha,\gamma} = \frac{1}{2}\cdot 2\,\frac{(-1)\,x^\gamma m}{r^3} = -\frac{m x^\gamma}{r^3}$$

$$\boxed{B_{\alpha\alpha,\gamma} = -\frac{m x^\gamma}{r^3}} \quad \boxed{\alpha \neq \gamma}$$

$$B_{\alpha\alpha,\alpha} = \frac{1}{2}\left[2\frac{\partial h_{\alpha\alpha}}{\partial x^\alpha} - \frac{\partial h_{\alpha\alpha}}{\partial x^\alpha} - \frac{\partial h_{\alpha\alpha}}{\partial x^\alpha}\right] = 0$$

$$\boxed{B_{\alpha\alpha,\alpha} = 0}$$

$\alpha \neq \beta$

$$B_{\alpha\beta,\alpha} = \frac{1}{2}\left[2\frac{\partial h_{\alpha\beta}}{\partial x^\alpha} - \frac{\partial h_{\alpha\alpha}}{\partial x^\beta} - \frac{\partial h_{\beta\alpha}}{\partial x^\alpha}\right]$$

$$B_{\alpha\beta,\alpha} = \frac{1}{2}\left[\frac{m x^\beta}{r^3}\right] = \frac{1}{2}\frac{m x^\beta}{r^3}$$

$$\boxed{B_{\alpha\beta,\alpha} = \frac{1}{2}\frac{m x^\beta}{r^3}} \quad \boxed{\alpha \neq \beta}$$

$B^i_{jk} = 0$

1) cuando los tres índices son desiguales,
2) cuando los tres índices son iguales,
3) cuando sólo un índice es igual a 1

~~$B^i_{jk} \neq 0$~~

~~1) cuando sólo dos índices son iguales, excepto $B^1_{\alpha\alpha}$ que es igual a cero.~~

~~$B^i_{jk} \neq 0$~~

~~1) cuando dos sólo dos índices son iguales a 1,~~

$B^i_{jk} \neq 0$

1) cuando sólo dos índices son iguales y el ~~tercer~~ índice restante no es igual a 1.

$$\cancel{B^1_{1\alpha} = B_{1\alpha,1} = \frac{m x^\alpha}{r^3}}$$

$$\cancel{B^1_{\alpha\alpha} = B_{\alpha\alpha,1} = - \frac{m x^\alpha}{r^3}}$$

$$\cancel{B^\alpha_{11} = - B_{11,\alpha} = \frac{m x^\alpha}{r^3}}$$

$$\cancel{B^1_{\alpha\beta} = B_{\alpha\beta,1} = 0 \quad \alpha \neq \beta}$$

$$\underline{\alpha \neq \beta, \ \beta \neq \gamma, \ \gamma \neq \alpha}$$

$$B^1_{11} = B_{11,1} = 0$$

$$B^1_{1\alpha} = B_{1\alpha,1} = \frac{1}{2} \frac{m x^\alpha}{r^3}$$

$$B^1_{\alpha\beta} = B_{\alpha\beta,1} = 0 \quad \alpha \neq \beta$$

$$\underline{B^1_{\alpha\alpha} = B_{\alpha\alpha,1} = 0}$$

$$B^\alpha_{11} = - B_{11,\alpha} = \frac{m x^\alpha}{r^3}$$

$$B^\alpha_{1\alpha} = - B_{1\alpha,\alpha} = 0 \qquad B^\alpha_{\beta\gamma} = - B_{\beta\gamma,\alpha} = 0$$

$$B^\alpha_{1\beta} = - B_{1\beta,\alpha} = 0$$

$$B^\alpha_{\alpha\alpha} = - B_{\alpha\alpha,\alpha} = 0$$

$$B^\alpha_{\alpha\beta} = - B_{\alpha\beta,\alpha} = - \frac{1}{2} \frac{m x^\beta}{r^3}$$

$$B^\alpha_{\beta\beta} = - B_{\beta\beta,\alpha} = \frac{m x^\alpha}{r^3}$$

$$B_{13,1} = \frac{1}{2}\left\{ \cancel{2\frac{\partial h_{13}}{\partial t}} - \frac{\partial h_{11}}{\partial y} - \cancel{\frac{\partial h_{31}}{\partial t}} \right\}$$

$$B_{13,1} = \frac{1}{2}\frac{m y}{r^3}$$

$$B^1_{1\alpha} = B_{1\alpha,1} = \frac{1}{2}\frac{mx^\alpha}{r^3}$$

$$B^\alpha_{11} = -B_{11,\alpha} = \frac{mx^\alpha}{r^3}$$

$$B^\alpha_{\alpha\beta} = -B_{\alpha\beta,\alpha} = -\frac{1}{2}\frac{mx^\beta}{r^3}$$

$$B^\alpha_{\beta\beta} = -B_{\beta\beta,\alpha} = \frac{mx^\alpha}{r^3}$$

Comprobaciones

$$B_{23,2} = \frac{1}{2}\left[2\frac{\partial h_{23}}{\partial x} - \frac{\partial h_{22}}{\partial y} - \frac{\partial h_{32}}{\partial x}\right]$$

$$B_{23,2} = +\frac{1}{2}\frac{my}{r^3}$$

$$B^2_{23} = -\frac{1}{2}\frac{my}{r^3}$$

$$\cancel{B^3_{44} = \frac{1}{2}\left\{2\frac{\partial h}{}\right.}$$

$$B_{44,3} = \frac{1}{2}\left\{2\frac{\partial h_{44}}{\partial y} - \frac{\partial h_{43}}{\partial z} - \frac{\partial h_{43}}{\partial z}\right\}$$

$$B_{44,3} = -\frac{my}{r^3}$$

$$B_{11,4} = \frac{1}{2}\left\{2\frac{\partial h_{11}}{\partial z} - \frac{\partial h_{14}}{\partial t} - \frac{\partial h_{14}}{\partial t}\right\}$$

$$B_{11,4} = -\frac{mz}{r^3}$$

$\cancel{B^{i}_{jk} B^{m}_{il} =}$

El tensor de pseudocurvatura P^{i}_{jkl} es;

$$P^{i}_{jkl} = B^{m}_{jl} B^{i}_{mk} - B^{m}_{jk} B^{i}_{ml} + \frac{\partial}{\partial x^k} B^{i}_{jl} - \frac{\partial}{\partial x^l} B^{i}_{jk}$$

$$P^{1}_{111} = \underbrace{B^{m}_{11} B^{1}_{m1} - B^{m}_{11} B^{1}_{m1}}_{0} + \underbrace{\frac{\partial}{\partial t} B^{1}_{11}}_{0} - \underbrace{\frac{\partial}{\partial t} B^{1}_{11}}_{0}$$

$$\boxed{P^{1}_{111} = 0} \qquad \boxed{P^{i}_{jkk} = 0}$$

$$P^{1}_{11\alpha} = B^{m}_{1\alpha} B^{1}_{m1} - B^{m}_{11} B^{1}_{m\alpha} + \underbrace{\frac{\partial}{\partial t} B^{1}_{1\alpha}}_{0} - \underbrace{\frac{\partial}{\partial x^\alpha} B^{1}_{11}}_{0}$$

$$P^{1}_{11\alpha} = \begin{cases} \cancel{B^{1}_{1\alpha} B^{1}_{11}} + \cancel{B^{2}_{1\alpha} B^{1}_{21}} + \cancel{B^{3}_{1\alpha} B^{1}_{31}} + \cancel{B^{4}_{1\alpha} B^{1}_{41}} \\ - \cancel{B^{1}_{11} B^{1}_{1\alpha}} - \cancel{B^{2}_{11} B^{1}_{2\alpha}} - \cancel{B^{3}_{11} B^{1}_{3\alpha}} - \cancel{B^{4}_{11} B^{1}_{4\alpha}} \end{cases}$$

$$P^{1}_{12} = \begin{cases} \cancel{B^{2}_{12} B^{1}_{21}} + \cancel{B^{2}_{12} B^{1}_{31}} + \cancel{B^{4}_{12} B^{1}_{41}} \\ - \cancel{B^{2}_{11} B^{1}_{22}} - \cancel{B^{3}_{11} B^{1}_{32}} - \cancel{B^{4}_{11} B^{1}_{42}} \end{cases}$$

$\cancel{P^{1}_{12}} = \cancel{-C\,\frac{m x}{r^3}}$. $\quad P^{1}_{112} = 0$

$$\boxed{P^{1}_{11\alpha} = 0}$$

$P^1_{123} =$

Simetrías

$$P^i_{jkl} = B^m_{jl} B^i_{mk} - B^m_{jk} B^i_{ml} + \frac{\partial}{\partial x^k} B^i_{jl} - \frac{\partial}{\partial x^l} B^i_{jk}$$

$$P^i_{jlk} = B^m_{jk} B^i_{ml} - B^m_{jl} B^i_{mk} + \frac{\partial}{\partial x^l} B^i_{jk} - \frac{\partial}{\partial x^k} B^i_{jl}$$

$$\boxed{P^i_{jkl} = - P^i_{jlk}}$$

$$P^i_{kjl} = B^m_{kl} B^i_{mj} - B^m_{kj} B^i_{ml} + \frac{\partial}{\partial x^j} B^i_{kl} - \frac{\partial}{\partial x^l} B^i_{kj}$$

$$P^i_{jkl} = B^m_{jl} B^i_{mk} - B^m_{jk} B^i_{ml} + \frac{\partial}{\partial x^k} B^i_{jl} - \frac{\partial}{\partial x^l} B^i_{jk}$$

$$P^i_{klj} = B^m_{kj} B^i_{ml} - B^m_{kl} B^i_{mj} + \frac{\partial}{\partial x^l} B^i_{kj} - \frac{\partial}{\partial x^j} B^i_{kl}$$

$$P^i_{ljk} = B^m_{lk} B^i_{mj} - B^m_{lj} B^i_{mk} + \frac{\partial}{\partial x^j} B^i_{lk} - \frac{\partial}{\partial x^k} B^i_{lj}$$

$P^1_{1jk} = 0 \qquad P^1_{1jj} = 0$

¡Todas las $P^1_{1..}$ son nulas!

P^1_1 ✓

P^1_{111} ✓ 0	P^1_{112} 0	P^1_{113} 0	P^1_{114} 0
P^1_{121} 0	P^1_{122} ✓ 0	P^1_{123}. 0	P^1_{124} 0
P^1_{131} 0	P^1_{132} 0	P^1_{133} ✓ 0	P^1_{134} 0
P^1_{141} 0	P^1_{142} 0	P^1_{143} 0	P^1_{144} ✓ 0

P'_2

P'_{211} 0	P'_{212} $\frac{m^2}{4r^6}\left[2(y^2+z^2)-x^2\right]$ $-\frac{m}{2}\frac{r^2-3x^2}{r^5}$	P'_{213} $\frac{3mxy}{4r^6}\boxed{2r-m}$	P'_{214} $\frac{3mxz}{4r^6}\left[2r-m\right]$
P'_{221} $-\frac{m^2}{4r^6}\left[2(y^2+z^2)-x^2\right]$ $+\frac{m}{2}\frac{r^2-3x^2}{r^5}$	P'_{222} 0	P'_{223} 0	P'_{224} 0
P'_{231} $\frac{3mxy}{4r^6}\left[m-2r\right]$	P'_{232} 0	P'_{233} 0	P'_{234} 0
P'_{241} $\frac{3mxz}{4r^6}\left[m-2r\right]$	P'_{242} 0	P'_{243} 0	P'_{244} 0

P'_3

P'_{311} 0	P'_{312} $\frac{3mxy}{4r^6}[2r-m]$	P'_{313} $\frac{m^2}{4r^6}[2(x^2+z^2)-y^2]$ $-\frac{m}{2}\frac{r^2-3y^2}{r^5}$	P'_{314} $\frac{3myz}{4r^6}[2r-m]$
P'_{321} $\frac{3mxy}{4r^6}[m-2r]$	P'_{322} 0	P'_{323} 0	P'_{324} 0
P'_{331} $-\frac{m^2}{4r^6}[2(x^2+z^2)-y^2]$ $+\frac{m}{2}\frac{r^2-3y^2}{r^5}$	P'_{332} 0	P'_{333} 0	P'_{334} 0
P'_{341} $\frac{3myz}{4r^6}[m-2r]$	P'_{342} 0	P'_{343} 0	P'_{344} 0

$$p'_4$$

p'_{411} 0	p'_{412} $\frac{3mxz}{4r^6}[2r-m]$	p'_{413} $\frac{3myz}{4r^6}[2r-m]$	p'_{414} $\frac{m^2}{4r^6}[2(x^2+y^2)-z^2]$ $-\frac{m}{2}\frac{r^2-3z^2}{r^5}$
p'_{421} $\frac{3mxz}{4r^6}[m-2r]$	p'_{422} 0	p'_{423} 0	p'_{424} 0
p'_{431} $\frac{3myz}{4r^6}[m-2r]$	p'_{432} 0	p'_{433} 0	p'_{434} 0
p'_{441} $-\frac{m^2}{4r^6}[2(x^2+y^2)-z^2]$ $+\frac{m}{2}\frac{r^2-3z^2}{r^5}$	p'_{442} 0	p'_{443} 0	p'_{444} 0

P_1^2

P^2_{111} 0	P^2_{112} $\frac{m}{2r^5}\left[mr-2r^2+6x^2\right]$	P^2_{113} $\cancel{\frac{3mxy}{2r^6}\left[m+2r\right]}$ $\frac{3mxy}{r^5}$	P^2_{114} $\cancel{\frac{3mxz}{2r^6}\left[m+2r\right]}$ $\frac{3mxz}{r^5}$
P^2_{121} $-\frac{m}{2r^5}\left[mr-2r^2+6x^2\right]$	P^2_{122} 0	P^2_{123} 0	P^2_{124} 0
P^2_{131} $\cancel{-\frac{3mxy}{2r^6}\left[m+2r\right]}$ $-\frac{3mxy}{r^5}$	P^2_{132} 0	P^2_{133} 0	P^2_{134} 0
P^2_{141} $\cancel{-\frac{3mxz}{2r^6}\left[m+2r\right]}$ $-\frac{3mxz}{r^5}$	P^2_{142} 0	P^2_{143} 0	P^2_{144} 0

P^2_2

P^2_{211} 0	P^2_{212} 0	P^2_{213} 0	P^2_{214} 0
P^2_{221} 0	P^2_{222} 0	P^2_{223} $\frac{3mxy}{4r^6}[2r-m]$	P^2_{224} $\frac{3mxz}{4r^6}[2r-m]$
P^2_{231} 0	P^2_{232} $-\frac{3mxy}{4r^6}[2r-m]$	P^2_{233} 0	P^2_{234}
P^2_{241} 0	P^2_{242} $-\frac{3mxz}{4r^6}[2r-m]$	P^2_{243}	P^2_{244} 0

P^2_3

P^2_{311} 0	P^2_{312}	P^2_{313}	P^2_{314}
P^2_{321}	P^2_{322} 0	P^2_{323}	P^2_{324}
P^2_{331}	P^2_{332}	P^2_{333} 0	P^2_{334}
P^2_{341}	P^2_{342}	P^2_{343}	P^2_{344} 0

P^2_4

P^2_{411} 0	P^2_{412}	P^2_{413}	P^2_{414}
P^2_{421}	P^2_{422} 0	P^2_{423}	P^2_{424}
P^2_{431}	P^2_{432}	P^2_{433} 0	P^2_{434}
P^2_{441}	P^2_{442}	P^2_{443}	P^2_{444} 0

$$P^3_1$$

p^3_{111} ✓ 0	p^3_{112}	p^3_{113}	p^3_{114}
p^3_{121}	p^3_{122} ✓ 0	p^3_{123}	p^3_{124}
p^3_{131}	p^3_{132}	p^3_{133} ✓ 0	p^3_{134}
p^3_{141}	p^3_{142}	p^3_{143}	p^3_{144} ✓ 0

P^3_2

P^3_{211} 0	P^3_{212}	P^3_{213}	P^3_{214}
P^3_{221}	P^3_{222} 0	P^3_{223}	P^3_{224}
P^3_{231}	P^3_{232}	P^3_{233} 0	P^3_{234}
P^3_{241}	P^3_{242}	P^3_{243}	P^3_{244} 0

P^3_3

P^3_{311} 0	P^3_{312}	P^3_{313} 0	P^3_{314}
P^3_{321}	P^3_{322} 0	P^3_{323}	P^3_{324}
P^3_{331} 0	P^3_{332}	P^3_{333} 0	P^3_{334}
P^3_{341}	P^3_{342}	P^3_{343}	P^3_{344} 0

P^3_4

P^3_{411} 0	P^3_{412}	P^3_{413}	P^3_{414}
P^3_{421}	P^3_{422} 0	P^3_{423}	P^3_{424}
P^3_{431}	P^3_{432}	P^3_{433} 0	P^3_{434}
P^3_{441}	P^3_{442}	P^3_{443}	P^3_{444} 0

p^4_1

p^4_{111} 0	p^4_{112}	p^4_{113}	p^4_{114}
p^4_{121}	p^4_{122} 0	p^4_{123}	p^4_{124}
p^4_{131}	p^4_{132}	p^4_{133} 0	p^4_{134}
p^4_{141}	p^4_{142}	p^4_{143}	p^4_{144} 0

P^4_2

P^4_{211} 0	P^4_{212}	P^4_{213}	P^4_{214}
P^4_{221}	P^4_{222} 0	P^4_{223}	P^4_{224}
P^4_{231}	P^4_{232}	P^4_{233} 0	P^4_{234}
P^4_{241}	P^4_{242}	P^4_{243}	P^4_{244} 0

p^4_3

p^4_{311} 0	p^4_{312}	p^4_{313}	p^4_{314}
p^4_{321}	p^4_{322} 0	p^4_{323}	p^4_{324}
p^4_{331}	p^4_{332}	p^4_{333} 0	p^4_{334}
p^4_{341}	p^4_{342}	p^4_{343}	p^4_{344} 0

p^4_4

p^4_{411} ✓ 0	p^4_{412}	p^4_{413}	p^4_{414} ✓ 0
p^4_{421}	p^4_{422} ✓ 0	p^4_{423}	p^4_{424}
p^4_{431}	p^4_{432}	p^4_{433} ✓ 0	p^4_{434}
p^4_{441} ✓ 0	p^4_{442}	p^4_{443}	p^4_{444} ✓ 0

$$\boxed{\dot{} = \frac{d}{dt}} \quad \boxed{{}' = \frac{d}{ds}}$$

$$\dot{x} = \frac{x'}{t'}$$

$$\ddot{x} = \frac{t'x'' - x't''}{t'^3}$$

$$x'' = \left(\frac{\partial h_{2j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x}\right)\frac{dx^j}{ds}\frac{dx^k}{ds}$$

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$$\ddot{x} = \left(\frac{\partial h_{2j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x}\right)\dot{x}^j\dot{x}^k - \dot{x}\left(\frac{\partial h_{1j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial t}\right)\dot{x}^j\dot{x}^k$$

$$\ddot{x} = \left[\frac{\partial h_{2j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x} - \dot{x}\left\{\frac{\partial h_{1j}}{\partial x^k} - \frac{\partial h_{jk}}{\partial t}\right\}\right]\dot{x}^j\dot{x}^k$$

$$\ddot{x} = \frac{\partial h_{21}}{\partial t} - \frac{\partial h_{11}}{\partial x} + \frac{\partial h_{21}}{\partial x^\alpha}\dot{x}^\alpha + \frac{\partial h_{2\alpha}}{\partial t}\dot{x}^\alpha + \frac{\partial h_{2\alpha}}{\partial x^\beta}\dot{x}^\alpha\dot{x}^\beta$$
$$- \cancel{\frac{\partial h_{11}}{\partial t}\dot{x}} - \frac{\partial h_{11}}{\partial x^\alpha}\dot{x}\dot{x}^\alpha - \cancel{\frac{\partial h_{1\alpha}}{\partial t}\dot{x}\dot{x}^\alpha} - \frac{\partial h_{1\alpha}}{\partial x^\beta}\dot{x}\dot{x}^\alpha\dot{x}^\beta$$
$$+ \cancel{\frac{\partial h_{11}}{\partial t}\dot{x}} + \cancel{\frac{\partial h_{1\alpha}}{\partial t}\dot{x}\dot{x}^\alpha} + \frac{\partial h_{\alpha 1}}{\partial t}\dot{x}\dot{x}^\alpha + \frac{\partial h_{\alpha\beta}}{\partial t}\dot{x}\dot{x}^\alpha\dot{x}^\beta$$

$$\ddot{x} = \left[\frac{\partial h_{21}}{\partial t} - \frac{\partial h_{11}}{\partial x}\right] + \left[\frac{\partial h_{21}}{\partial x^\alpha} + \frac{\partial h_{2\alpha}}{\partial t}\right]\dot{x}^\alpha$$
$$+ \left[-\frac{\partial h_{11}}{\partial x^\alpha} + \frac{\partial h_{\alpha 1}}{\partial t}\right]\dot{x}\dot{x}^\alpha + \frac{\partial h_{2\alpha}}{\partial x^\beta}\dot{x}^\alpha\dot{x}^\beta + \left[\frac{\partial h_{\alpha\beta}}{\partial t} - \frac{\partial h_{1\alpha}}{\partial x^\beta}\right]\dot{x}\dot{x}^\alpha\dot{x}^\beta$$